双曲扁壳式閘門应力計算

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§1　引言

在一般建筑上应用双曲扁壳是属于簡支而且是均布荷重的情形，但双曲扁壳式閘門则是非簡支非均布荷重的情形，本文任务是对此情形提出应力計算方程及公式，由于我們都是沒有学过薄壳理論，这次是边学边干，所以难免存在很多錯誤，希望各界人士严加指正，我們不胜感謝。

在叙述前先說明本文所采用的符号如下，

A_1, A_2——拉梅系数；R_1, R_2——曲率半径；$\varepsilon_1, \varepsilon_2$——伸长应变；$\varepsilon$——切变；$v, v_2$，$w$——位移函数；$D$——抗变刚度；$f_a, f_b$——扁壳的矢高。至于其他符号的意义和一般力学书上的相同。

§2　問題假設

一般薄壳問題的方程很复杂，往往对具体問題要作一系列假設，才把方程簡化，当然这些做法要保証在允許的誤差范围之內，現在我們对問題作如下假設：

1. 以扁壳中面的主曲率綫作曲綫坐标系，可近地似用平面笛氏坐标系代替。则有 $ds_1 = A_1 d_1 X$，$ds_2 = A_2 d_2 X$. ∵ $dx \approx d_1 X$，$dy \approx d_2 X$，∴ $A_1 = A_2 = 1$.

2. $S = T_{12} = T_{21}$.

3. $\frac{N_1}{R_1}$，$\frac{N_2}{R_2}$ 可忽略不計。

4. 在曲率改变率 K_1，K_2 和扭率 L 公式中忽略 $\frac{u}{R_1}$，$\frac{v}{R_2}$，$\frac{1}{R_1}\frac{\partial u}{\partial y}$，$\frac{1}{R_2}\frac{\partial u}{\partial x}$.

5. 忽略 $\frac{1}{R_1 R_2}$ 同阶項。

§2　建立方程

根据平衡方程，連續性方程、变形方程①以及假設1—5得：

$$\frac{\partial T_1}{\partial x} + \frac{\partial s}{\partial y} + q_1 = 0 \qquad \cdots\cdots\cdots(1)$$

$$\frac{\partial s}{\partial x} + \frac{\partial T_2}{\partial y} + q_2 = 0 \qquad \cdots\cdots\cdots(2)$$

$$\frac{\partial M_1}{\partial x} + \frac{\partial M_{21}}{\partial y} - N_1 = 0 \qquad \cdots\cdots\cdots(3)$$

$$\frac{\partial M_1}{\partial x}+\frac{\partial M_2}{\partial y}-N_2=o \quad \cdots\cdots\cdots(4)$$

$$\frac{K_1}{R_2}+\frac{K_2}{R_1}+\frac{\partial^2\varepsilon_2}{\partial x^2}+\frac{\partial^2\varepsilon_1}{\partial y^2}-\frac{\partial^2\omega}{\partial x\partial y}=o\cdots\cdots\cdots(5)$$

$$K_1=-\frac{\partial^2 w}{\partial x^2},\ K_2=-\frac{\partial^2 w}{\partial y^2},\ L=-\frac{\partial^2 w}{\partial x\partial y}.$$

另外从薄壳理論还知道：$M_1=D(k_1+\mu k_2)$，$M_{21}=\frac{E\delta_3}{12(1+\mu)}L$.

则$M_1=-D\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right)$，$M_{21}=-D(1-\mu)\frac{\partial^2 w}{\partial x\partial y}$，$M_1$、$M_{21}$代入(3)(4)得：

$N_1=-D\frac{\partial}{\partial x}(\Delta w)$，$N_2=-D\frac{\partial}{\partial y}(\Delta w)$，又将$N_1$、$N_2$代入平衡方程之三式9得：

$$\frac{T_1}{R_1}+\frac{T_2}{R_2}+D\Delta\Delta(w)=q_n \quad \cdots\cdots\cdots(6)$$

从問題的提法知$q_1=o$，而q_2是自重所产生的力，而$q_2=-r$(单位体重)q_n則是水压力，設水面至薄壳上頂之高度为h，則$q_n=y+h$。

引入应力函数φ并使$T_1=\frac{\partial^2\varphi}{\partial y^2}$，$T_2=\frac{\partial^2\varphi}{\partial x^2}-\gamma y$，$S=T_{21}=T_{12}=-\frac{\partial^2\varphi}{\partial x\partial y}$. 显然满足(1)(2)，而(6)变为：$\Delta_k\varphi+D\Delta\Delta W=h+y+\frac{\gamma y}{R_2}\cdots\cdots\cdots(7)$

从而中面的变形和内力的关系如下：

$$\left.\begin{aligned}\varepsilon_1&=\frac{1}{E\delta}(T_1-\mu T_2)=\frac{1}{E\delta}\left(\frac{\partial^2\varphi}{\partial y^2}-\mu\frac{\partial^2\varphi}{\partial x^2}+\mu\gamma y\right)\\ \varepsilon_2&=\frac{1}{E\delta}(T_2-\mu T_1)=\frac{1}{E\delta}\left(\frac{\partial^2\varphi}{\partial x^2}-\mu\frac{\partial^2\varphi}{\partial y}-\gamma y\right)\end{aligned}\right\}\cdots\cdots\cdots(8),$$

$$\omega=\frac{2(1+\mu)}{E\delta}S=-\frac{2(1+\mu)}{E\delta}\frac{\partial^2\varphi}{\partial x\partial y}.$$

从而有$\frac{\partial^2\varepsilon_1}{\partial y^2}+\frac{\partial^2\varepsilon_2}{\partial x^2}-\frac{\partial^2\omega}{\partial x\partial y}=\frac{1}{E\delta}\Delta\Delta\varphi\cdots\cdots\cdots(9)$，将(8)(9)代入(5)得：

$$\frac{1}{E\delta}\Delta\Delta\Phi-\Delta_k w=o\cdots\cdots\cdots(10)$$

現得所得之方程列于下：

$$\begin{cases}\Delta_k\varphi+D\Delta\Delta w=h+y+\frac{\gamma y}{R_2} & \cdots\cdots\cdots(7)\\ \Delta\Delta\varphi-E\delta\Delta_k w=o & \cdots\cdots\cdots(10)\end{cases}$$

則应力由下列公式求出：

$T_1=\frac{\partial^2\varphi}{\partial y^2}$，$T_2=\frac{\partial^2\varphi}{\partial x^2}-\gamma y$，$J=T_{21}=T_{12}=-\frac{\partial^2\varphi}{\partial x\partial y}$；$M_1=-D\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right)$，

$M_2=-D\left(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}\right)$，$M=-\frac{E\delta^3}{12(1+\mu)}\frac{\partial^2 w}{\partial x\partial y}$.

上面 $\Delta_k=\frac{1}{R_1}\frac{\partial^2}{\partial y^2}+\frac{1}{R_2}\frac{\partial^2}{\partial x^2}$；$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$；$\gamma$——单位体积壳体重量。

§边界条件

1.支承情况　双曲扁壳是垂直放在水底，故一边受重力作用。而曲面受到垂直于面的水压力，为了簡单起见不計算水对四边的压力，此外，还有一个問題要考虑，就是 $x=o$，$x=a$时，由于閘門的自重作用在oy方向产生位移v，为了使$v=o$,故必須在边界上有一应力$T_2=-\gamma y$，抵消自重的作用。

2.边界条件

a）$x=o$. $x=a$. $T_1=M_1=V=W=o$，$T_2=-\gamma y$，则可推得$\frac{\partial^2\varphi}{\partial y^2}=o$，

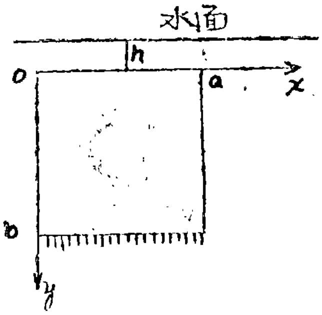
$\frac{\partial^2\varphi}{\partial x^2}=o$，$\frac{\partial^2 w}{\partial x^2}=-\mu\frac{\partial^2 w}{\partial y^2}$.

b）$y=o$. $T_2=M_2=U=W=O$，这样可得$\frac{\partial^2\varphi}{\partial x^2}=o$，$\frac{\partial^2\varphi}{\partial y^2}=o$，

$\frac{\partial^2 w}{\partial y^2}=-\mu\frac{\partial^2 w}{\partial x^2}$.

C）$y=b$. $M_2=u=w=o$，$T_2=-\gamma b$，同样得 $\frac{\partial^2\varphi}{\partial x^2}=o$，$\frac{\partial^2\varphi}{\partial y^2}=o$，

$\frac{\partial^2 w}{\partial y^2}=-\mu\frac{\partial^2 w}{\partial x^2}$.



§3 問題解法

用双三角級数求解，令：

$$\varphi(z,y)=\sum_{m=1}^{\infty}\sum_{=1}^{\infty}B_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{q}y\cdots\cdots\cdots(2)$$

$$w(x,y)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\mathrm{Sin}\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\cdots\cdots(3)$$

显然满足所有的边界条件。

設 $L_{mn}^{(1)}=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2$，$L_{mn}^{(2)}=\left[\frac{1}{R_1}\left(\frac{n\pi}{b}\right)^2+\frac{1}{R_2}\left(\frac{m\pi}{a}\right)^2\right]$，将（2）（3）代入（1）之第二式得 $B_{mn}=\frac{-E\delta L_{mn}^{(2)}A_{mn}}{L_{mn}^{(1)}}\cdots\cdots(4)$

又将（1）之第一式的右边展成重三角級数：

$$h+y+\frac{\gamma y}{p_2}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}c_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\cdots\cdots(5)$$

可求出$c_{2m,2n}=c_{2m,2n-1}=o$；$c_{2m-1,2n}=\frac{-4b(R_2+\gamma)}{R_2n(2m-1)\pi^2}$；$c_{2m-1,2n-1}=\frac{16R_2b+8b(R_2+\gamma)}{(2m-1)(2n-1)R_2\pi^2}$

将（2）（3）（5）代入（1）一式得：$DL_{mn}^{(1)}A_{mn}-L_{mn}^{(1)}B_{mn}=c_{mn}\cdots\cdots(6)$

∴从(1)(6)得: $A_{mn}=\dfrac{L_{mn}^{(1)}c_{mn}}{DL_{mn}^{(1)^2}+E\delta L_{mn}^{(2)^2}}$ ……(7)

$$B_{mn}=\frac{-E\delta L_{mn}^{(2)}c_{mn}}{DL_{mn}^{(1)^2}+E\delta L_{mn}^{(2)^2}}\cdots\cdots(8)$$

这样应力依下列公式求出:

$$T_1=\frac{\partial^2\varphi}{\partial y^2}=-\left(\frac{\pi}{b}\right)^2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}n^2B_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\quad\cdots\cdots\cdots\cdots(9)$$

$$T_2=\frac{\partial^2\varphi}{\partial x^2}-\gamma y=-\left(\frac{\pi}{a}\right)^2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m^2B_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y-\gamma y\cdots\cdots(10)$$

$$S=-\frac{\partial^2\varphi}{\partial x\partial y}=\frac{\pi^2}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m\,n\,B_{mn}\cos\frac{m\pi}{a}x\cos\frac{n\pi}{b}y\quad\cdots\cdots\cdots\cdots(11)$$

$$M_1=-D\left(\frac{\partial^2w}{\partial x^2}+\mu\frac{\partial^2w}{\partial y^2}\right)=\frac{D\pi^2}{a^2b^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}(b^2m^2+\mu a^2n^2)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{d}y\cdots(12)$$

$$M_2=-D\left(\frac{\partial^2w}{\partial y^2}+\mu\frac{\partial^2w}{\partial x^2}\right)=\frac{D\pi^2}{a^2b^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}(a^2n^2+\mu b^2m^2)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\cdots(13)$$

$$H=-\frac{E\delta^3}{12(1+\mu)}\frac{\partial^2w}{\partial x\partial y}=-\frac{D\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m\,n\,A_{mn}\cos\frac{m\pi}{a}x\cos\frac{n\pi}{b}y\quad\cdots\cdots\cdots(14)$$

式中Amn及Bmn由(7)(8)决定。

双曲扁壳式閘門的具体数据如下:

$a=2.9m$, $b=3.4m$, $fa=fb=0.5m$, $\delta=10cm$, $E=38\times10^4kg/cm^2$, $\mu=0$ 。

曲面方程: $z=fa+fb-\dfrac{4fa}{a^2}\left(\dfrac{a}{2}-x\right)^2-\dfrac{4fb}{b^2}\left(\dfrac{b}{2}-y\right)^2$; $k_1=\dfrac{1}{R_1}=\dfrac{8fa}{a^2}$, $k_2=\dfrac{1}{R_2}=\dfrac{8fb}{b^2}$ 。

$D=\dfrac{E\delta^3}{12(1-\mu^2)}$。将这些数据代入(9)——(14)并取m=1, 3; n=1, 2得:

$$T_1=0.853\Big(8201.231\sin\frac{\pi}{2.9}x\sin\frac{\pi}{3.4}y-2098.636\sin\frac{\pi}{2.9}x\sin\frac{2\pi}{3.4}y$$
$$+298.271\sin\frac{3\pi}{2.9}x\sin\frac{\pi}{3.4}y-81.704\sin\frac{3\pi}{2.9}x\sin\frac{2\pi}{3.4}y\Big)$$

$$T_2=1.173\Big(8201.231\sin\frac{\pi}{2.9}x\sin\frac{\pi}{3.4}y-524.659\sin\frac{\pi}{2.9}x\sin\frac{2\pi}{3.4}y$$
$$+2684.139\sin\frac{3\pi}{2.9}x\sin\frac{\pi}{3.4}y-183.834\sin\frac{3\pi}{2.9}x\sin\frac{2\pi}{3.4}y\Big)-2400y.$$

$$S=8201.231\cos\frac{\pi}{2.9}x\cos\frac{\pi}{3.9}y-1049.317\cos\frac{\pi}{2.9}x\cos\frac{2\pi}{3.4}y$$
$$+894.814\cos\frac{3\pi}{2.9}x\cos\frac{\pi}{3.4}y-122.556\cos\frac{3\pi}{2.9}x\cos\frac{2\pi}{3.4}y$$

$$M_1=44.235\times10^{-2}\Big(12706.37\sin\frac{\pi}{2.9}x\cos\frac{\pi}{3.4}-1671.507\sin\frac{\pi}{2.9}x\sin\frac{2\pi}{3.4}y$$
$$+22195.185\sin\frac{3\pi}{2.9}x\sin\frac{\pi}{3.4}y-2076.534\sin\frac{3\pi}{2.9}x\sin\frac{2\pi}{3.4}y\Big)$$

$$M_2=44.235\times10^{-2}\Big(9243.994\sin\frac{\pi}{2.9}x\sin\frac{\pi}{3.4}y-4864.142\sin\frac{\pi}{2.9}x\sin\frac{2\pi}{3.4}y$$
$$+1794.13\sin\frac{3\pi}{2.9}x\sin\frac{\pi}{3.4}y-671.421\sin\frac{3\pi}{2.9}x\sin\frac{2\pi}{3.4}y\Big)$$

$$H=3.17\times10^{-2}\Big(-1099.167\cos\frac{\pi}{2.9}x\cos\frac{\pi}{3.4}y+289.118\cos\frac{\pi}{2.9}x\cos\frac{\pi}{3.4}y$$
$$-639.999\cos\frac{3\pi}{2.9}x\cos\frac{\pi}{3.4}y+119.754\cos\frac{3\pi}{2.9}x\cos\frac{2\pi}{3.4}y\Big)$$

§5 应力分布情况

应力＼点	(o,o)	$(o,\frac{b}{2})$	(o,b)	$(\frac{a}{2},o)$	(a,o)	$(a,\frac{b}{2})$	$(\frac{a}{2},b)$	(a,b)
T_1	0	0	0	0	0	0	0	0
T_2	0	−4.408	−8.160	0	0	−4.080	−8.160	−8.160
S	7.923	1.192	−10.268	0	−7.923	−1.192	0	10.268

应力＼点	$(\frac{3}{4}a,\frac{b}{2})$	$(\frac{a}{2},\frac{3}{4}b)$	$(\frac{a}{4},\frac{b}{4})$	$(\frac{a}{2},\frac{b}{2})$	$(\frac{a}{4},\frac{b}{2})$	$(\frac{a}{2},\frac{b}{4})$	$(\frac{3}{4}a,\frac{3}{4}b)$
T_1	5.100	6.484	2.302	6.725	5.100	4.975	4,940
T_2	4.955	−3.121	3.757	2.374	4.955	2.091	0.852
S	−0.655	0	3.653	0	0.655	0	3.653

表中应力单位：T / m²

①见　　薄壳理論

Calculation for the stress of double curved flat-shell type water-gate

Faculty of Mathematic-Mechanic Class four

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There are broad application of hyperbolic flat-shell in cioil engineering but must of them are of simple cendition of simple support and distributed load. Now the double curved flat-shell type water-gate is neither simple support nor distributed load. This article propose the equations and for mule for calculating this problem in cluding the following contents

1. hypothesis problem
2. equation established
3. boundary condition
4. Calcululing formule and resuet