常系数二阶两个自变数两个未知函数的綫性椭圓型微分方程組(II)

——特征四次型有一对复重根的情况

林　偉　吳兹潛

（数学力学系）

摘　要

本文研究了特征方程有一对复重根的常系数二阶两个自变数两个未知函数的綫性椭圓型微分方程組

$$\begin{pmatrix}1&0\\0&\lambda\end{pmatrix}\frac{\partial^2}{\partial x^2}\begin{pmatrix}u\\v\end{pmatrix}+\begin{pmatrix}0&\lambda-1\\\lambda-1&0\end{pmatrix}\frac{\partial^2}{\partial x\partial y}\begin{pmatrix}u\\v\end{pmatrix}+\begin{pmatrix}\lambda&0\\0&1\end{pmatrix}\frac{\partial^2}{\partial y^2}\begin{pmatrix}u\\v\end{pmatrix}=0,\quad(\lambda\neq0,1)$$

的 Neumann 問題等三类边界問題的解的存在唯一性。

§1　引　言

設A,B,C是三个二行二列的实数方陣，則矩陣形式的偏微分方程

$$A\frac{\partial^2}{\partial x^2}\begin{pmatrix}u\\v\end{pmatrix}+2B\frac{\partial^2}{\partial x\partial y}\begin{pmatrix}u\\v\end{pmatrix}+C\frac{\partial^2}{\partial y^2}\begin{pmatrix}u\\v\end{pmatrix}=0\tag{1.1}$$

是两个自变数x,y,，两个未知函数u,v的两个方程所成的常系数二阶綫性偏微分方程組。

行列式　$Q(\xi,\eta)=|A\xi^2+2B\xi\eta+C\eta^2|$　定义为偏微分方程組(1.1)的特征四次型。如果(1.1)的特征四次型的根全为复根，則称(1.1)为椭圓型的。在我們的文章[1]中已証明，如果(1.1)的特征四次型有一对复重根时，它經过自变数的綫性变换，未知函数的綫性变换及方程之間的綫性組合等三种变换后，可化为如下的标准型

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$$\begin{pmatrix}1&0\\0&\lambda\end{pmatrix}\frac{\partial^2}{\partial x^2}\begin{pmatrix}u\\v\end{pmatrix}+2\begin{pmatrix}0&\frac{\lambda-1}{2}\\\frac{\lambda-1}{2}&0\end{pmatrix}\frac{\partial^2}{\partial x\partial y}\begin{pmatrix}u\\v\end{pmatrix}+\begin{pmatrix}\lambda&0\\0&1\end{pmatrix}\frac{\partial^2}{\partial y^2}\begin{pmatrix}u\\v\end{pmatrix}=0 \quad (1.2)$$

这儿 λ 是实数，$\lambda\neq 0,1$。

命

$$x+iy=z,\qquad u+iv=f(z),$$

$$\frac{1}{2}\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)=\frac{\partial}{\partial z},\qquad \frac{1}{2}\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)=\frac{\partial}{\partial \bar{z}},$$

我們在〔2〕中已証明(1.2)有如下的复数形式

$$(1-\lambda)\frac{\partial^2 f}{\partial z^2}+(1+\lambda)\frac{\partial^2 \bar{f}}{\partial z\partial \bar{z}}=0 \quad (1.3)$$

及一般解

$$f(z)=\frac{\lambda-1}{4\lambda}\bar{z}\varphi(z)+\frac{\lambda+1}{4\lambda}\overline{\int_0^z\varphi(\xi)d\xi}+\psi(z), \quad (1.4)$$

这儿 $\varphi(z)$ 及 $\psi(z)$ 是 z 的解析函数。

在文章〔1〕，〔2〕我們已分别証明，当 $\lambda>0$ 时，(1.2)的 Dirichlet 問題的解是唯一的和存在的。本文将研究方程組(1.2)的另外三类边界問題解的存在性及唯一性，（参考§2，§3，§4）。

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§2 第二問題(Neu mann問題)

第二問題是求函数 u,v，使它們在单位圓內适合方程組(1.2)及边界条件：

$$\left.\begin{aligned}\frac{\partial u}{\partial n}\Big|_{|z|=1}&=g(\zeta)\\\frac{\partial v}{\partial n}\Big|_{|z|=1}&=h(\zeta)\end{aligned}\right\} \quad (2.1)$$

这儿 $\zeta=e^{it}$，$g(\zeta)$，$h(\zeta)$ 是給定在单位圓 $|\zeta|=1$ 上的函数，其微商 $g'(\zeta)$，$h'(\zeta)$ 适合Hölder条件，而且

$$(\lambda+1)\frac{1}{2\pi}\int_0^{2\pi}[g(e^{it})+ih(e^{it})]dt$$

$$=(\lambda-1)\frac{1}{2\pi}\int_0^{2\pi}2[g(e^{it})-ih(e^{it})]e^{-i2t}dt。 \quad (2.2)$$

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我們用 Fourier 分析的方法来解决这問題，这就是从方程組的一般解(1.4)出发，命

$$\varphi(z)=\sum_{n=0}^{\infty}a_nz^n,\qquad \psi(z)=\sum_{n=0}^{\infty}b_nz^n,\qquad z=\rho e^{i\theta},\tag{2.3}$$

卽得

$$f(z)=\frac{\lambda-1}{4\lambda}\bar{z}\sum_{n=0}^{\infty}a_nz^n+\frac{\lambda+1}{4\lambda}\sum_{n=0}^{\infty}\frac{\bar{a}_n}{n+1}\bar{z}^{n+1}+\sum_{n=0}^{\infty}b_nz^n,$$

$$\frac{\partial f}{\partial n}=\frac{\partial f}{\partial\rho}=\sum_{n=0}^{\infty}\left[\frac{\lambda-1}{4\lambda}(n+2)a_{n+1}\rho^{n+1}+nb_n\rho^{n-1}\right]e^{in\theta}$$

$$+\left(\frac{\lambda-1}{4\lambda}a_0+\frac{\lambda-1}{4\lambda}\bar{a}_0\right)e^{-i\theta}+\sum_{n=2}^{\infty}\left(\frac{\lambda+1}{4\lambda}\right)\bar{a}_{n+1}\rho^{n-1}e^{-in\theta},$$

命 $g(\zeta)+ih(\zeta)=\omega(\zeta)$，則由条件(2.1)得

$$\omega(\zeta)=\frac{\partial f}{\partial n}\Big|_{|z|=1}=\sum_{n=0}^{\infty}\left[\frac{\lambda-1}{4\lambda}(n+2)a_{n+1}+nb_n\right]e^{in\theta}$$

$$+\left(\frac{\lambda-1}{4\lambda}a_0+\frac{\lambda+1}{4\lambda}\bar{a}_0\right)e^{-i\theta}+\sum_{n=2}^{\infty}\frac{\lambda+1}{4\lambda}\bar{a}_{n-1}e^{-in\theta}$$

立得

$$\frac{\lambda+1}{4\lambda}\bar{a}_{n-1}=\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)e^{int}dt,\qquad n\geqslant 2,\tag{2.4}$$

$$\frac{\lambda-1}{4\lambda}a_0+\frac{\lambda+1}{4\lambda}\bar{a}_0=\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)e^{it}dt,\tag{2.5}$$

$$\frac{\lambda-1}{4\lambda}(n+2)a_{n+1}+nb_n=\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)e^{-int}dt,\qquad n\geqslant 0,\tag{2.6}$$

当 $\lambda\neq-1$ 时，卽得

$$a_n=\frac{1}{2\pi}\int_0^{2\pi}\frac{4\lambda}{\lambda+1}\overline{\omega(\zeta)}e^{-i(n+1)t}dt,\qquad n\geqslant 1,\tag{2.7}$$

由(2.6)当$n=0$时得

$$\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)dt=\frac{\lambda-1}{\lambda+1}\frac{1}{2\pi}\int_0^{2\pi}2\overline{\omega(\zeta)}e^{-i2t}dt$$

这正是条件(2.2)，当$n\geqslant 1$时得

$$b_n=\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)\frac{e^{-int}}{n}dt-$$

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$$-\frac{1}{2\pi}\int_0^{2\pi}\frac{\lambda-1}{\lambda+1}\overline{\omega(\zeta)}\left(1+\frac{2}{n}\right)e^{-i(n+2)t}dt,\quad (n\geqslant 1)\qquad (2.8)$$

将(2.7)(2.8)代入(2.3)得

$$\varphi(z)=a_0+\frac{1}{2\pi}\int_0^{2\pi}\frac{4\lambda}{\lambda+1}\overline{\omega(\zeta)}\frac{ze^{-it}}{1-ze^{-it}}e^{-it}dt\ ,\qquad (2.9)$$

$$\int_0^z\varphi(\xi)d\xi=a_0z-\frac{1}{2\pi}\int_0^{2\pi}\frac{4\lambda}{\lambda+1}\overline{\omega(\zeta)}\left[ln(1-ze^{-it})+ze^{-it}\right]dt\ ,\qquad (2.10)$$

$$\psi(z)=b_0+\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)\left[-ln(1-ze^{-it})\right]dt$$

$$-\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\lambda-1}{\lambda+1}\right)\overline{\omega(\zeta)}\left[\frac{ze^{-it}}{1-ze^{-it}}-2ln(1-ze^{-it})\right]e^{-i2t}dt\qquad (2.11)$$

将(2.9)(2.10)代入并(1.4)并注意及(2.5)得

$$f(z)=b_0-\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)\left[ln(1-ze^{-it})+\overline{ln(1-ze^{-it})}\right]dt$$

$$+\frac{1}{2\pi}\int_0^{2\pi}\frac{\lambda-1}{\lambda+1}\overline{\omega(\zeta)}\left[\overline{\frac{z(z-e^{it})}{1-ze^{-it}}}+2ln(1-ze^{-it})\right]e^{-i2t}dt\ ,\qquad (2.12)$$

这便是問題2的解了，这儿b_0是任意复常数。

定理1 假設$g(\zeta)$，$h(\zeta)$是給定在单位圓$|z|=1$上的函数，其微商滿足HÖlder条件，且

$$\frac{1}{2\pi}\int_0^{2\pi}\left[g(e^{it}+ih(e^{it})\right]dt=\left(\frac{\lambda-1}{\lambda+1}\right)\frac{1}{2\pi}\int_0^{2\pi}\left[g(e^{it})-\right.$$

$$\left.-ih(e^{it})\right]e^{-i2t}dt\ ,\qquad (2.1)$$

則方程組(1.2)当$\lambda\neq 0$，± 1，它存在唯一解(2.12)滿足如下的边界条件

$$\left.\frac{\partial u}{\partial n}\right|_{|z|=1}=g(\zeta),\qquad \left.\frac{\rho v}{\partial n}\right|_{|z|=1}=h(\zeta)$$

这儿所謂唯一性是指容許函数u、v差一常数項.

証. 1)解(2.12)滿足方程組(1.2)

由(2.12) 得

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$$\frac{\partial^2 f}{\partial \bar{z}^2} = \frac{1}{2\pi}\int_0^{2\pi}\omega(e^{it})\left(\overline{\frac{e^{-it}}{1-ze^{-it}}}\right)^2 dt \quad , \tag{2.13}$$

$$\frac{\partial^2 f}{\partial \bar{z}\partial z} = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\lambda-1}{\lambda+1}\right)\omega(e^{it})\left(\overline{\frac{e^{-it}}{1-ze^{-it}}}\right)^2 dt \tag{2.14}$$

将(2.13)，(2.14)代入(13)得

$$(1-\lambda)\frac{\partial^2 f}{\partial \bar{z}^2} + (1+\lambda)\frac{\partial^2 f}{\partial z\partial \bar{z}} = \left[(1-\lambda)+(\lambda-1)\right]\frac{1}{2\pi}\int_0^{2\pi}\omega(e^{it})\left(\overline{\frac{e^{-it}}{1-ze^{-it}}}\right)^2 dt = 0$$

因此解(2.12)适合(1.3)，(2.12)即所对应的 u、v 滿足(1.2)

2)解(2.12)滿足边界条件

由(2.12)得

$$f(z) = b_0 - \frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)ln\left(1-2\rho cos(\theta-t)+\rho^2\right)dt$$

$$+\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\lambda-1}{\lambda+1}\right)\overline{\omega(\zeta)}\left[\frac{\rho^2-\rho e^{i(\theta-t)}}{1-\rho e^{i(\theta-t)}} + 2\,ln(1-\rho e^{i(\theta-t)})\right]e^{-i2t}dt \ ,$$

故得

$$\frac{\partial f}{\partial n} = \frac{\partial f}{\partial \rho} = \frac{1}{\rho}\left\{\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)\left[\frac{1-\rho^2}{1-2\rho cos(\theta-t)+\rho^2}-1\right]dt\right.$$

$$+\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\lambda-1}{\lambda+1}\right)\overline{\omega(\zeta)}\left[\frac{2\rho^2-\rho e^{i(\theta-t)}-\rho^3 e^{i(\theta-t)}}{(1-\rho e^{i(\theta-t)})^2} + \right.$$

$$\left.\left.+\frac{2\rho e^{i(\theta-t)}}{1-\rho e^{i(\theta-t)}}\right]e^{-i2t}dt\right\} ,$$

由条件(2.2)及恒等式

$$\frac{2\rho^2-\rho e^{i(\theta-t)}-\rho^3 e^{i(\theta-t)}}{(1-\rho e^{i(\theta-t)})^2} + \frac{2\rho e^{i(\theta-t)}}{1-\rho e^{i(\theta-t)}} - 2 = \frac{(\rho^2-1)(2-ze^{-it})}{(1-ze^{-it})^2}$$

故得

$$\frac{\partial f}{\partial n} = \frac{1}{\rho}\left\{\frac{1}{2\pi}\int_0^{2\pi}\omega(t)\frac{1-\rho^2}{1-2\rho cos(\theta-t)+\rho^2}dt\right.$$

$$+(\rho^2-1)\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\lambda-1}{\lambda+1}\overline{\omega(\zeta)}\left[\frac{2e^{it}-z}{(e^{it}-z)^2}e^{-it}\right]dt\right. \tag{2.15}$$

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由于$\omega'(\zeta)$满足Hölder条件，因此按照Сохоцкий公式[3]，下面极限

$$\lim_{\rho\to 1}\frac{1}{2\pi}\int_0^{2\pi}\overline{\omega(\zeta)}\,\frac{2e^{it}-z}{(e^{it}-z)^2}\,e^{-it}dt$$

$$=\lim_{z\to e^{i\theta}}\frac{1}{2\pi i}\int_{|z|=1}\frac{\overline{\omega'(\zeta)\zeta^3}}{(\zeta-z)}\,d\zeta$$

存在．由(2.15)得

$$\frac{\partial f}{\partial n}\Big|_{|z|=1}=\lim_{\rho\to 1}\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)\frac{1-\rho^2}{1-2\rho\cos(\theta-t)+\rho^2}\,dt$$

$$+\lim_{\rho\to 1}(\rho^2-1)\left(\frac{\lambda-1}{\lambda+1}\right)\frac{1}{2\pi}\int_0^{2\pi}\omega(\zeta)\left[\frac{2e^{it}-z}{(e^{it}-z)^2}-e^{it}\right]dt$$

$$=\omega(\zeta)=g(\zeta)+ih(\zeta).$$

因此解(2.12)滿足边界条件(2.2)，明所欲証。

§3 第三問題(1)及(2)

第三問題(1)是求函数u，v，使它們在单位圓$|z|<1$內适合方程組(1.2)，并滿足如下的边界条件：

$$u(z)\Big|_{|z|=1}=g(\zeta),\tag{3.1}$$

$$\frac{\partial u}{\partial n}\Big|_{|z|=1}=h(\zeta),\tag{3.2}$$

这儿$g(\zeta)$，$h(\zeta)$是給定在$|\zeta|=1$上的連續函数。

仿照§2，我們从(1.2)的一般解(1.4)出发，利用Fourier方法，得出这个問題的复数解为

$$f(z)=b_0+\frac{\lambda-1}{4\lambda}a_1\rho^2+\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\frac{2ze^{-it}}{(1-ze^{-it})}\,dt-(\rho^2-1)\left[\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\right.$$

$$\left.\cdot\frac{ze^{-it}}{(1-ze^{-it})^2}\,dt-\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)\,\frac{ze^{-it}}{1-ze^{-it}}\,dt\right]+i2Im\left\{-\left(\frac{\lambda+1}{4\lambda}a_0\right.\right.$$

$$\left.+\frac{\lambda-1}{4\lambda}\bar{a}_0\right)z-\frac{\lambda+1}{8\lambda}a_1z^2+\left(\frac{\lambda+1}{\lambda-1}\right)\left[\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\left(ze^{-it}+\frac{ze^{-it}}{1-ze^{-it}}\right.\right.$$

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$$+ 2ln(1-ze^{-it})\Big) e^{i2t}dt + \frac{1}{2\pi}\int_0^{2\pi} h(\zeta)\Big(ze^{-it} + \frac{(ze^{-it})^2}{2}$$

$$+ ln(1-ze^{-it})\Big) e^{i2t}\,dt\Big]\Big\}, \tag{3.3}$$

这儿$b_0=\frac{1}{2\pi}\int_0^{2\pi}(g(\zeta)-\frac{1}{2}h(\zeta))\,dt+ic_0$，$a_0=c_1+ic_2$，$a_1=\frac{2\lambda}{\lambda-1}\Big[\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)dt$ $+i2c_3\Big]$，而c_0，c_1，c_2，c_3是任意实常数，即得

$$u(z)=(\rho^2-1)^2\Big[\frac{1}{2\pi}\int_0^{2\pi}\Big(-\frac{h(e^{it})}{2}\frac{dt}{1-2\rho cos(\theta-t)+\rho^2}$$

$$+\frac{1}{2\pi}\int_0^{2\pi} g(e^{it})\frac{1-\rho\rho cos(\theta-t)}{[1-2\rho cos(\theta-t)+\rho^2]^2}\,dt\Big], \tag{3.4}$$

$$v(z)=c_0+Im\Big\{-\Big(\frac{\lambda+1}{2\lambda}a_0+\frac{\lambda-1}{2\lambda}\bar{a}_0\Big)z-\Big(\frac{\lambda+1}{2\lambda}\Big)a_1z^2+\frac{\lambda-1}{4\lambda}a_1\rho^2$$

$$+\frac{1}{2\pi}\int_0^{2\pi} g(\zeta)\frac{2ze^{-it}}{1-ze^{-it}}\,dt-(\rho^2-1)\Big[\frac{1}{2\pi}\int_0^{2\pi} g(\zeta)\frac{ze^{-it}}{(1-ze^{-it})^2}\,dt$$

$$-\frac{1}{2\pi}\int_0^{2\pi} h(\zeta)\frac{ze^{-it}}{1-ze^{it}}\,dt\Big]+\Big(\frac{\lambda+1}{\lambda-1}\Big)\Big[\frac{1}{2\pi}\int_0^{2\pi} 2g(\zeta)\Big(ze^{-it}$$

$$+\frac{ze^{-it}}{1-ze^{-it}}+2ln(1-ze^{-it})\Big)e^{2it}dt+\frac{1}{2\pi}\int_0^{2\pi} 2h(\zeta)\Big(ze^{-it}$$

$$+\frac{(ze^{-it})^2}{2}+ln(1-ze^{-it})\Big)e^{i2t}\,dt\Big]\Big\}, \tag{3.5}$$

定理2　假設$g(\zeta)$及$h(\zeta)$是給定在单位圓$|z|=1$上的連續函数，則方程組(1.2)当$\lambda\neq 0$，1时，在单位圓$|z|<1$內存在唯一解(3.4)、(3.5)滿足如下的边界条件：

$$u(z)\Big|_{|z|=1}=g(\zeta),\qquad \frac{\partial u}{\partial n}\Big|_{|z|=1}=h(\zeta),$$

这里所謂唯一性是指容許函数$v(z)$差一个二次項$c_0+c_1y+c_2x+c_3(x^2-y^2)$

証. 解(3.4)(3.5)的复数形式是(3.3)，由(3.3)得

$$\frac{\partial^2 f}{\partial\bar{z}^2}=\frac{\lambda+1}{4\lambda}\bar{a}_1-\Big(\frac{\lambda+1}{\lambda-1}\Big)\Big\{\frac{1}{2\pi}\int_0^{2\pi} g(\zeta)\overline{\Big(\frac{2ze^{-i2t}}{(1-ze^{-it})^3}\Big)}\,dt$$

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$$+\frac{1}{2\pi}\int_0^{2\pi} h(\zeta)\left[1-\overline{\left(\frac{1}{(1-ze^{-it})^2}\right)}\right]dt \Bigg\}, \tag{3.6}$$

$$\frac{\partial^2 f}{\partial \bar{z}\partial z}=\frac{\lambda-1}{4\lambda}\bar{a}_1-\left\{\frac{1}{2\pi}\int_0^{2\pi} g(\zeta)\overline{\left(\frac{2ze^{-i2t}}{(1-ze^{-it})^3}\right)}dt\right.$$

$$\left.+\frac{1}{2\pi}\int_0^{2\pi} h(\zeta)\left[1-\overline{\left(\frac{1}{(1-ze^{-it})^2}\right)}\right]dt\right\}, \tag{3.7}$$

将(3.6)，(3.7)代入(1.3)得

$$(1-\lambda)\frac{\partial^2 f}{\partial \bar{z}^2}+(1+\lambda)\frac{\lambda^2\bar{f}}{\partial \bar{z}\partial z}=0$$

即(3.3)满足方程(1.3)，因此解(3.4)，(3.5)满足方程組(1.2)。解(3.4)也是满足条件(3.1)(3.2)的，这可仿照解双調和方程的相应边界問題的方法証明之。

用同样方法，我們得如下問題的解：

定理3.（第三問題(2)）。假設$g(\zeta)$及$h(\zeta)$是給定右单位圓$|\zeta|=1$上的函数，其微商$g''(\zeta)$及$h'(\zeta)$在$|\zeta|=1$上滿足 Hölder 条件，則方程組(1.2)当$\lambda<0$时，它在单位圓$|z|<1$內存在唯一解

$$u(z)=\frac{1}{2\pi}\int_0^{2\pi} g(e^{it})\,Re\left(\frac{e^{it}+z}{e^{it}-z}\right)dt+\frac{\lambda-1}{4\lambda}(z\bar{z}-1)$$

$$Re\left(\frac{\varphi(\zeta)-(c_1+ic_2)}{z}\right), \tag{3.8}$$

$$v(z)=\frac{1}{2\pi}\int_0^{2\pi} g(e^{it})\,Im\left(\frac{e^{it}+z}{e^{it}-z}\right)dt$$

$$+\frac{\lambda-1}{4\lambda}(z\bar{z}-1)\,Im\left(\frac{\varphi(z)-(c_1+ic_2)}{z}\right)-2\left(\frac{\lambda+1}{4\lambda}\right)Im\Big(\Phi(z)\Big)$$

$$-\left(\frac{\lambda-1}{2\lambda}\right)(c_1\rho\sin\theta-c_2\rho\cos\theta)+\frac{1}{2\pi}\int_0^{2\pi}\tfrac{1}{2}h(e^{it})dt+c_4, \tag{3.9}$$

滿足如下边界条件：

$$u(z)\Big|_{|z|=1}=g(\zeta), \tag{3.10}$$

$$\frac{\partial v}{\partial n}\Big|_{|z|=1}=h(\zeta), \tag{3.11}$$

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这儿出数 $\Phi(z)=\int_0^z \varphi(\xi)d\xi$，而 $\varphi(z)$是如下的函数：当$\lambda=-1$时，

$$\varphi(z)=\left[\left(c_1+ic_2\right)+\left(c_3+i\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}h(e^{it})dt\right)z+\ +(c_1-ic_2)z^2\right]$$

$$+\frac{1}{2\pi}\int_0^{2\pi}\frac{4\lambda}{\lambda-1}ih(e^{it})\frac{z^2e^{-it}}{1-ze^{-it}}dt-\frac{1}{2\pi}\int_0^{2\pi}\frac{4\lambda}{\lambda-1}g(e^{it})\frac{z^2e^{-it}}{(1-ze^{-it})^2}dt \quad (3.12)$$

当$\lambda\neq-1$时

$$\varphi(z)=\frac{-(c_1+ic_2)}{k^2}+\frac{z}{1-k^2z^2}\Big[(c_1+ic_2)+\frac{1}{k^2}(c_1-ic_2)\ +$$

$$+\left(c_3+i\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}h(e^{it})dt\right)\Big]$$

$$+\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{4\lambda}{\lambda-1}\right)\left(ih(e^{it})\right)\frac{z^2e^{-it}}{(1-ze^{-it})(1-k^2z^2)}dt$$

$$-\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{4\lambda}{\lambda-1}\right)g(e^{it})\frac{z^2e^{-it}}{(1-ze^{-it})^2(1-k^2z^2)}dt \quad (3.13)$$

其中 c_1,c_2,c_3,c_4，是任意实常数，$k^2=\frac{\lambda+1}{\lambda-1}$，$(|k|<1)$.

証：解(3.8)，(3.9)可写为如下的复数形式：

$$f(z)=\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\frac{\zeta+z}{\zeta-z}dt-\frac{\lambda+1}{4\lambda}\left(\Phi(z)-\overline{\Phi(z)}\right)+$$

$$+\frac{\lambda-1}{4\lambda}(z\bar{z}-1)\frac{\varphi(z)-a_0}{z}-\frac{\lambda-1}{4\lambda}(\bar{a}_0z-a_0\bar{z})+$$

$$+i\left(\frac{1}{2\pi}\int_0^{2\pi}\frac{h(\zeta)}{2}dt+c_4\right), \quad (3.14)$$

这儿 $a_0=c_1+i\ c_2=\varphi(o)$，$\zeta=e^{it}$　易証(3.14)适合(1.3)，因此(3.8)(3.9)滿足方程組(1.2)．現在証明解(3.8)(3.9)适合边界条件。

由(3.12)，(3.13)，注意及$g'(\zeta)$滿足Hölder条件，故

$\lim\limits_{z\to\zeta_0}\left(\frac{\varphi(z)-a_0}{z}\right)$ 存在[3]，由(3.8)得

$$u(z)\Big|_{(z)=1}=\lim_{z\to\zeta_0}\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)Re\left(\frac{\zeta+z}{\zeta-z}\right)dt$$

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$$+\lim_{z\to\zeta_0}\left[\frac{\lambda-1}{4\lambda}(z\bar{z}-1)\,Re\left(\frac{\psi(z)-a_0}{z}\right)\right]=g(\zeta_0)$$

即(3.8)适合(3.10). 由(3.9)得

$$\frac{\partial v}{\partial n}=\frac{\partial v}{\partial\rho}=\frac{2}{\rho}\,Im\left\{\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\,\frac{\zeta z}{(\zeta-z)^2}\,dt+\right.$$

$$\left.+\frac{\lambda-1}{4\lambda}\,\frac{\varphi(z)}{z}\left(\rho^2-\frac{\lambda+1}{\lambda-1}z^2\right)\right\}-\frac{2}{\rho}\,\frac{\lambda-1}{4\lambda}\,Im\left(\bar{a}_0z+a_0\frac{\rho^2}{z}\right)+$$

$$+\frac{\lambda-1}{4\lambda}(\rho^2-1)\;Im\left[\frac{d}{dz}\left(\frac{\varphi(z)-a_0}{z}\right)\frac{z}{\rho}\right],\qquad(3.15)$$

注意及

$$\frac{\lambda-1}{4\lambda}\,\frac{\varphi(z)}{z}\left(\rho^2-\frac{\lambda+1}{\lambda-1}z^2\right)=\frac{\lambda-1}{4\lambda}\,\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\left(\frac{a_0}{z}+\bar{a}_0z+c_3\right)$$

$$+\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\left[\frac{1}{2\pi}\int_0^{2\pi}ih(\zeta)\frac{\zeta+z}{2(\zeta-z)}\,dt-\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\,\frac{\zeta z}{(\zeta-z)^2}\,dt\right]$$

代入(3.15)得

$$\frac{\partial v}{\partial n}=\frac{1}{\rho}\,Im\left[\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\,\frac{1}{2\pi}\int_0^{2\pi}ih(\zeta)\,\frac{\zeta+z}{\zeta-z}\,dt\right]$$

$$+\frac{2}{\rho}\,Im\left[\frac{1-\rho^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\,\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\,\frac{\zeta z}{(\zeta-z)^2}\,dt\right]$$

$$+\frac{2}{\rho}\,Im\left[\frac{\lambda-1}{4\lambda}\,\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\left(\frac{a_0}{z}+\bar{a}_0z+c_3\right)\right]-$$

$$-\frac{2}{\rho}\,Im\left[\frac{\lambda-1}{4\lambda}\left(a_0\frac{\rho^2}{z}+\bar{a}_0z\right)\right]+$$

$$+\frac{\lambda-1}{4\lambda}\left(\rho^2-1\right)Im\left[\frac{d}{dz}\left(\frac{\varphi(z)-a_0}{z}\right)\frac{z}{\rho}\right],$$

令 $\rho\to1$，由于$g''(\zeta)$及$h'(\zeta)$满足Hölder条件，因此 $\lim\limits_{\rho\to1}\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\,\frac{\zeta z}{(\zeta-z)^2}\,dt$ 及

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$\lim\limits_{\rho\to 1}\dfrac{d}{dz}\left(\dfrac{\varphi(z)-a_0}{z}\right)$ 存在[3]，又因

$$\lim_{\rho\to 1}\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}=1,\quad \lim_{\rho\to 1}\frac{1-\rho^2}{1-\frac{\lambda+1}{\lambda-1}z^2}=0$$

$$\lim_{\rho\to 1} Im\left[\frac{\lambda-1}{4\lambda}\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\left(\frac{a_0}{z}+\bar{a}_0 z+c_3\right)\right]$$

$$=\frac{\lambda-1}{4\lambda}Im\left(\bar{a}_0 e^{i\theta}+a_0 e^{i\theta}\right)=0$$

$$\lim_{\rho\to 1} Im\left[\frac{\lambda-1}{4\lambda}\left(a_0\frac{\rho^2}{z}+\bar{a}_0 z\right)\right]=Im\left[\frac{\lambda-1}{4\lambda}\left(a_0 e^{-i\theta}+\bar{a}_0 e^{i\theta}\right)\right]=0$$

故得

$$\left.\frac{\partial v}{\partial n}\right|_{|z|=1}=\lim_{\rho\to 1}\frac{1}{\rho}Im\left[\left(\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\right)\frac{1}{2\pi}\int_0^{2\pi}ih(\zeta)\frac{\zeta+z}{\zeta-z}dt\right]$$

$$+\lim_{\rho\to 1}\frac{2}{\rho}Im\left[\frac{1-\rho^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\frac{\zeta z}{(\zeta-z)^2}dt\right]+$$

$$+\lim_{\rho\to 1}\frac{2}{\rho}Im\left[\frac{\lambda-1}{4\lambda}\cdot\frac{\rho^2-\frac{\lambda+1}{\lambda-1}z^2}{1-\frac{\lambda+1}{\lambda-1}z^2}\left(\frac{a_0}{z}+\bar{a}_0 z+c_3\right)\right]-$$

$$-\lim_{\rho\to 1}\frac{2}{\rho}Im\left[\frac{\lambda-1}{4\lambda}\left(a_0\frac{\rho^2}{z}+\bar{a}_0 z\right)\right]$$

$$+\frac{\lambda-1}{4\lambda}\lim_{\rho\to 1}(\rho^2-1)\,Im\left[\frac{d}{dz}\left(\frac{\varphi(z)-a_0}{z}\right)\frac{z}{\rho}\right]$$

$$=\lim_{\rho\to 1}\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)Re\left(\frac{\zeta+z}{\zeta-z}\right)dt=h(\zeta),$$

即解(3.9)也适合边界条件(3.11)　明所欲証，

§4　第四問題(1)及(2)

第四問題(1)是求出数 u,v，它們在单位圓 $|z|<1$ 内适合方程組(1.2)，而且滿足如下的边界条件：

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$$u(z)\Big|_{|z|=1}=g(\zeta) \tag{4.1}$$

$$\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\Big|_{|z|=1}=h(\zeta), \tag{4.2}$$

这儿$g(\zeta)$是定义在$|\zeta|=1$上的連續函数，$h(\zeta)$在$|\zeta|=1$上滿足Hölder条件

我們从一般解(1.4)出发，利用解析函数的Schwartz公式，先由条件(4.1)，(4.2)求出函数$\varphi(z)$及$\psi(z)$，

由(1.4)有

$$Re\left(\frac{\partial f}{\partial \bar{z}}\right.=Re\left(\frac{\lambda-1}{4\lambda}\varphi(z)+\frac{\lambda+1}{4\lambda}\overline{\varphi(z)}\right)=\tfrac{1}{2}Re(\varphi(z)),$$

故由(4.2)有

$$Re(\varphi(z))\Big|_{|z|=1}=Re\left(2\frac{\partial f}{\partial \bar{z}}\right)_{|z|=1}=\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\Big|_{|z|=1}=h(\zeta),$$

由Schwartz公式得

$$\varphi(z)=\frac{1}{2\pi i}\int_{|z|=1}h(\zeta)\frac{\zeta+z}{\zeta-z}\frac{d\zeta}{\zeta}+iI_m(\varphi(0))$$

$$=\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)\frac{2z}{\zeta-z}dt+\varphi(0) \tag{4.3}$$

$$\int_0^z\varphi(\xi)d\xi=-\left(\frac{1}{2\pi i}\int_{|\zeta|=1}h(\zeta)ln(1-\frac{z}{\zeta})^2d\zeta+z\overline{\varphi(0)}\right) \tag{4.4}$$

由(1.4)及条件(4.2)得

$$Re(\psi(z))\Big|_{|z|=1}=g(\zeta)-\frac{\lambda-1}{4\lambda}Re(\bar{\zeta}\varphi(\zeta))-\frac{\lambda+1}{4\lambda}Re\left(\overline{\int_0^z\varphi(\xi)d\xi}\right)$$

按Schwartz公式故得

$$\psi(z)=\frac{1}{2\pi i}\int_{|\zeta|=1}g(\zeta)\frac{\zeta+z}{\zeta-z}\frac{d\zeta}{\zeta}-\frac{\lambda-1}{4\lambda}\frac{1}{2\pi i}$$

$$\int_{|\zeta|=1}Re\left(\frac{\varphi(\zeta)}{\zeta}\right)\frac{\zeta+z}{\zeta-z}\frac{d\zeta}{\zeta}-\frac{\lambda+1}{4\lambda}\int_0^z\varphi(\xi)d\xi+iIm(\psi(0)), \tag{4.5}$$

注意及$\psi(0)=f(0)$，立得

$$\psi(z)=\frac{1}{2\pi i}\int_{|\zeta|=1}g(\zeta)\frac{\zeta+z}{\zeta-z}\frac{d\zeta}{\zeta}-\frac{\lambda-1}{4\lambda}\left[\frac{\varphi(z)-\varphi(0)}{z}+\overline{\varphi(0)}z\right]$$

$$-\frac{\lambda+1}{4\lambda}\int_0^z\varphi(\xi)d\xi+iIm\left(f(0)+\frac{\lambda-1}{4\lambda}\varphi'(0)\right) \tag{4.6}$$

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将(4.3)，(4.4)，(4.6代入(1.4)得

$$f(z)=\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\frac{\zeta+z}{\zeta-z}dt-(1-z\bar{z})\frac{\lambda-1}{4\lambda}\frac{1}{2\pi}\int_0^{2\pi}\frac{2h(\zeta)}{\zeta-z}dt$$

$$+\frac{\lambda+1}{\lambda}iIm\left[\frac{1}{2\pi i}\int_{|z|=1}h(\zeta)ln\left(1-\frac{z}{\zeta}\right)d\zeta\right]+\frac{1}{2\lambda}(z\overline{\varphi(0)}-\bar{z}\varphi(0))$$

$$+iIm(f(0)+\frac{\lambda-1}{4\lambda}\varphi'(0)),\tag{4.7}$$

由(4.3)得

$$Im\ (\varphi'(0))=\frac{1}{2\pi}\int_0^{2\pi}(-2h(e^{it})sint)dt\ ,$$

如果命$Im(\varphi(0))=c_1$，　$Im(f(0))=v(0)=c_2$，由(4.7)得

$$f(z)=\frac{1}{2\pi}\int_0^{2\pi}g(\zeta)\frac{\zeta+z}{\zeta-z}dt-(1-z\bar{z})\frac{\lambda-1}{4\lambda}\frac{1}{2\pi}\int_0^{2\pi}\frac{2h(\zeta)}{\zeta-z}dt$$

$$+\frac{\lambda+1}{\lambda}iIm\left[\frac{1}{2\pi i}\int_{|z|=1}h(\zeta)ln(1-\frac{z}{\zeta}d\zeta\right]+\frac{i}{\lambda}Im\left[z\left(\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)dt\right.\right.$$

$$\left.\left.-ic_1)\right]+i\left[c_2+\frac{\lambda-1}{4\lambda}\frac{1}{2\pi}\int_0^{2\pi}(-2h(e^{it}))sintdt\right]\tag{4.8}$$

这就是問題四(1)的复数解了，由此可得

$$u(z)=\frac{1}{2\pi}\int_0^{2\pi}g(e^{it})\frac{1-\rho^2}{1-2\rho cos(\theta-t)+\rho^2}dt$$

$$-(1-\rho^2)\frac{\lambda-1}{2\lambda}\frac{1}{2\lambda}\int_0^{2\pi}h(e^{it})\frac{cost-\rho cos\theta}{1-2\rho cos\theta+\rho^2}dt,\tag{4.9}$$

$$v(z)=\frac{1}{2\pi}\int_0^{2\pi}g(e^{it})\frac{2\rho sin(\theta-t)}{1-2\rho cos(\theta-t)+\rho^2}dt+(1-\rho^2)\frac{\lambda-1}{2\lambda}\frac{1}{2\pi}\int_0^{2\pi}h(e^{it})$$

$$\frac{sint-\rho sin\theta}{1-2\rho cos(\theta-t)+\rho^2}dt+\frac{\lambda+1}{\lambda}\frac{1}{2\pi}\int_0^{2\pi}h(e^{it})\left[(ln|1-ze^{-it}|)sint\right.$$

$$\left.+(arg(1-ze^{-it}))cost\right]dt+\frac{1}{\lambda}\left[\left(\frac{1}{2\pi}\int_0^{2\pi}h(e^{it})dt\right)\rho sint\right.$$

$$\left.-C_1\rho cos\theta\right]+\left[C_2-\frac{\lambda-1}{2\lambda}\frac{1}{2\pi}\int_0^{2\pi}h(e^{it})sintdt\right]\tag{4.10}$$

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定理4 假設$g(\zeta)$，$h(\zeta)$是給定在$|\zeta|=1$上的連續函数，而且$g(\zeta)$在$|\zeta|=1$上滿足Hölder件条，則方程組(1.2)当$\lambda\neq 0$时，它在单位圓$|z|<1$內存在唯一解(4.9)(4.10)滿足边界条件：

$$u(z)\Big|_{|z|=1}=g(\zeta),\quad \frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\Big|_{|z|=1}=h(\zeta),$$

这儿所謂唯一性是指容許v差一个一次項(c_1+c_2x).

証. 由(4.8)易証$f(z)$滿足(1.3)，因之解(4.9)，(4.10)滿足方程組(1.2).現在証明u，v适合边界条件(4.1)(4.2).

由(4.9)注意及$h(\zeta)$滿足Hölder条件我們得

$$u(z)\Big|_{|z|=1}=\lim_{\rho\to 1}\Big[\frac{1}{2\pi}\int_0^{2\pi}g(e^{it})\frac{1-\rho^2}{1-2\rho\cos(\theta-t)+\rho^2}dt$$

$$+(1-\rho^2)\frac{\lambda-1}{4\lambda}\frac{1}{2\pi}\int_0^{2\pi}h(e^{it})\frac{\cos t-\rho\cos\theta}{1-2\rho\cos(\theta-t)+\rho^2}dt\Big]=g(\zeta),$$

由(4.8)得

$$\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=2Re\left(\frac{\partial f}{\partial \bar z}\right)$$

$$=2Re\Big\{\frac{\lambda-1}{4\lambda}\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)\frac{2z}{\zeta-z}dt+\frac{\lambda+1}{4\lambda}\overline{\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)\frac{2\zeta}{\zeta-z}dt}$$

$$-\frac{1}{2\lambda}\cdot\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)dt\Big\}=\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)Re\left(\frac{\zeta+z}{\zeta-z}\right)dt$$

故得

$$\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\Big|_{|z|=1}=\lim_{\rho\to 1}\frac{1}{2\pi}\int_0^{2\pi}h(\zeta)Re\left(\frac{\zeta+z}{\zeta-z}\right)dt=h(\zeta),$$

即u，v滿足边界条件(4.2)，明所欲証.

用类似方法我們可得：

定理5（第四問題(2)） 假設$g(\zeta)$，$h(\zeta)$是給定在$|\zeta|=1$上的連續函数，而且$h(\zeta)$在$|\zeta|=$上滿足Hölder条件，則方程組(1.2)当$\lambda\neq 0$，1时，它在单位圓$|z|<1$內存在唯一解：

$$u(z)=\frac{1}{2\pi}\int_0^{2\pi}g(e^{it})\frac{1-\rho^2}{1-2\rho\cos(\theta-t)+\rho^2}dt+(1-\rho^2)\frac{\lambda-1}{2\lambda}\frac{1}{2\pi}\int_0^{2\pi}(\lambda h(e^{it}))$$

$$\cdot\frac{\sin t-\rho\sin\theta}{1-2\rho\cos(\theta-t)+\rho^2}dt$$

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$$v(z)=\frac{1}{2\pi}\int_0^{2\pi} g(e^{it})\frac{2\rho sin(\theta-t)}{1-2\rho cos(\theta-t)+\rho^2}dt+(1-\rho^2)\frac{\lambda-1}{2\lambda}\frac{1}{2\pi}\int_0^{2\pi}(\lambda h(e^{it}))$$

$$\cdot\frac{cost-\rho cos\theta}{1-2\rho cos(\theta-t)+\rho^2}dt+\frac{\lambda+1}{\lambda}\frac{1}{2\pi}\int_0^{2\pi}(\lambda h(e^{it}))\Big[ln|1-ze^{-it}|cost$$

$$-\alpha ry(1-ze^{-it})sint\Big]dt+\frac{1}{\lambda}\Big[c_1\rho sin\theta+\Big(\frac{1}{2\pi}\int_0^{2\pi}(-\lambda)h(e^{it})dt\Big)$$

$$\cdot\rho cos\theta+\Big[c_1c_2-\frac{\lambda-1}{2\lambda}\frac{1}{2\pi}\int_0^{2\pi}(\lambda h(e^{it})cost)dt\Big]$$

满足如下的边界条件

$$u(z)\Big|_{|z|=1}=g(\zeta),\quad \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\Big|_{|z|=1}=h(\zeta),$$

这儿 c_1，c_2 是任意实常数。

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Системы линейных дифференциальных уравнений второго порядка эллиптического типа с постоянными коэффициентами(II)

Линь Вэй У Цзы-цянь

Резюме

В настоящей статье мы докажем существование и единственность решений трёх рода краевых задачей систем уравнений с частными прочзводпыми

$$\begin{pmatrix}1 & 0\\ 0 & \lambda\end{pmatrix}\frac{\partial^2}{\partial x^2}\begin{pmatrix}u\\ v\end{pmatrix}+\begin{pmatrix}0 & \lambda-1\\ \lambda-1 & 0\end{pmatrix}\frac{\partial^2}{\partial x\partial y}\begin{pmatrix}u\\ v\end{pmatrix}+\begin{pmatrix}\lambda & 0\\ 0 & 1\end{pmatrix}\frac{\partial^2}{\partial y^2}\begin{pmatrix}u\\ v\end{pmatrix}=0, \qquad (1)$$

здесь λ——вещественное, $\lambda\neq 0, 1$. Системы (1)——канонический вид систем уравнений второго порядка эллиптического типа с постоянными коэффициентами, характеристическое уравнение которых имеет два кратных комплексных корня.

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