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用U变换法分析薄壁箱形连续梁的组合扭转*

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摘 要 用U变换法分析具有周期性支承的薄壁箱形连续梁的组合扭转. 连续梁的控制方程经U变换解耦后，建立了子结构规模的控制方程和边界条件，并结合具体载荷条件，给出了位移和内力相应的解析表达式.

关键词 U变换，组合扭转，双力矩

1 基本方程

具均匀抗弯刚度$E'I_{\hat{\omega}a}$与抗扭刚度GI_k的n跨薄壁箱形连续梁(图1(a))，其子结构跨长为l.为形成等价的旋转周期系统，以原连续结构右外端支承为中心，结构对称、荷载反对称拓展；并使延拓结构末端与原结构始端的支承条件一致(令两端的广义扭率、双力矩相等)见图1(b)。于子结构上建立局部坐标系，设原点位于跨中点处，在局部坐标中建立的扭转微分方程为

$$d^4\phi_k(x)/dx^4-\lambda^2d^2\phi_k(x)/dx^2=\mu/E'I_{\hat{\omega}a}\cdot m_k(x)$$

$$\left(-\frac{l}{2}<x<\frac{l}{2},\ k=1,2\cdots,N\right) \quad (1)$$

其中$N=2n$，$\phi_k(x)$为扭角位移函数，λ为截面弯扭特性系数($\lambda^2=\mu GI_k/E'I_{\hat{\omega}a}$)，$\mu$为截面翘曲系数，$I_{\hat{\omega}a}$为截面广义主扇性惯矩，$m_k(x)$为第$k$子结构的荷载函数。

方程(1)应满足下列条件

$$\left.\begin{array}{l}\phi_k|_{x=l/2}=\phi_k|_{x=-l/2}=0\\ \dot{\theta}_k|_{x=l/2}=\dot{\theta}_{k+1}|_{x=-l/2}{}^{[1]}\\ B_{wk}|_{x=l/2}=B_{wk+1}|_{x=-l/2}\end{array}\right\} \quad (2)$$

其中$\dot{\theta}_k$为广义扭率，B_{wk}为双力矩。

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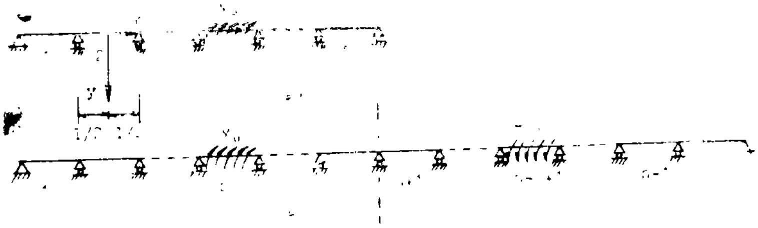

图1　作用均布扭矩薄壁箱形连续梁

Fig.1　Thin walled box girder with periodic supports to a distributed torque of constant intensity M_0

(a) 实际体系；　(b) 等效体系

由周期结构的旋转性质，方程(1)还应满足约束条件

$$\left.\begin{aligned}&\phi_N|_{x=l/2}=\phi_1|_{x=-l/2}\\&\dot{\theta}_N|_{x=l/2}=\dot{\theta}_1|_{x=-l/2}\\&B_{wN}|_{x=l/2}=B_{w1}|_{x=-l/2}\end{aligned}\right\}\tag{3}$$

将方程(1)，式(2)和(3)写成矩阵形式

设$\{\phi\}=[\phi_1\phi_2\cdots\phi_N]^T$；　$\{m\}=[m_1m_2\cdots m_N]^T$

则有

$$d^4\{\phi\}/dx^4-\lambda^2d^2\{\phi\}/dx^2=\mu/E'I_{\hat{w}a}\{m\}\tag{4}$$

和

$$\left.\begin{aligned}&\{\phi\}|_{x=l/2}=\{\phi\}|_{x=-l/2}=\{0\}\\&\{\phi'\}\Big|_{x=l/2}+\frac{1}{\mu k^2}\{\phi'''\}\Big|_{x=l/2}=[\varepsilon]^0\left[\{\phi'\}\Big|_{x=-l/2}+\frac{1}{\mu k^2}\{\phi'''\}\Big|_{x=-l/2}\right]\\&\beta E'I_{\hat{w}a}\{\phi''\}\Big|_{x=l/2}+\frac{1}{k^2}\{m\}\Big|_{x=l/2}=[\varepsilon]^0\left[\beta E'I_{\hat{w}a}\{\phi''\}\Big|_{x=-l/2}+\frac{1}{k^2}\{m\}\Big|_{x=-l/2}\right]\end{aligned}\right\}\tag{5}$$

其中，$k^2=\dfrac{\mu GI_\rho}{E'I_{\hat{w}a}}$，　$\beta=\dfrac{1}{\mu}$

$$[\varepsilon]^0=\begin{bmatrix}0&1&&&\\&0&1&&\\&\vdots&\ddots&\ddots&1\\1&0&&\ddots&0\end{bmatrix}_{N\times N}$$

为解除(5)连续条件耦连，需进行下列线性变换[2~4]

设　$$\{\phi\}=[U]\{q\}\tag{6}$$

$$\left.\begin{aligned}&\{q\}=[q_1\,q_2\cdots q_N]^T\\&[U]=[U_1\,U_2\cdots U_N]\end{aligned}\right\}\tag{7}$$

$[U]$的矢分量为

$$\{U_k\}=N^{-1/2}[1\quad\exp(ik\phi)\ \exp(i2k\phi)\cdots\exp(i(N-1)k\phi)]^T\tag{8}$$

其中，$\phi=2\pi/N$，$i=\sqrt{-1}$，$\{q\}$的分量$q_k(k=1,2,\cdots,N)$称广义位移(或旋转坐标)。

$[U]$有下列特性[2]：

$$[\bar{U}]^T[U]=[I] \quad (9)$$

$$[\bar{U}]^T[\varepsilon]^0[U]=\lceil e^{i\psi} \ e^{i2\psi}\cdots e^{iN\psi} \rfloor \quad (10)$$

其中，$[\bar{U}]^T$表示$[U]$的复共轭转置矩阵，$[I]$为单位矩阵，$[U]$满足式(9)称U矩阵，而式(6)则称U变换。

将(7)和(8)代入(6)则得

$$\phi_k=\frac{1}{\sqrt{N}}\sum_{m=1}^{N}\exp[i(k-1)m\phi]q_m, \qquad (k=1,\ 2,\ \cdots N) \quad (11)$$

这里(11)与(6)是等同的。

将(6)代入(4)和(5)并两端左乘$[\bar{U}]^T$得

$$d^4\{q\}/dx^4-\lambda^2d^2\{q\}/dx^2=\{f\} \quad , \quad \left(-\frac{l}{2}<x<\frac{l}{2}\right) \quad (12)$$

和

$$\left.\begin{aligned}&\{q\}|_{x=l/2}=\{q\}|_{x=-l/2}=\{O\}\\&\{q'\}\Big|_{x=l/2}+\frac{1}{\mu k^2}\{q'''\}\Big|_{x=l/2}=[\bar{U}]^T[\varepsilon]^0[U]\left[\{q'\}\Big|_{x=-l/2}+\frac{1}{\mu k^2}\{q'''\}\Big|_{x=-l/2}\right]\\&\{q''\}\Big|_{x=l/2}+\frac{1}{k^2}\{f\}\Big|_{x=l/2}=[\bar{U}]^T[\varepsilon]^0[U]\left[\{q''\}\Big|_{x=-l/2}+\frac{1}{k^2}\{f\}\Big|_{x=-l/2}\right]\end{aligned}\right\} \quad (13)$$

其中，$\{f\}=[\bar{U}]^T\dfrac{\mu}{E'I_{\hat{w}a}}\{m\}$

由(12)、(13)则得N组单一子结构的微分方程

$$d^4q_m/dx^4-\lambda^2d^2q_m/dx^2=f_m(x), \qquad \left(-\frac{l}{2}<x<\frac{l}{2},\ m=1,\ 2,\ \cdots,\ N\right) \quad (14)$$

和

$$\left.\begin{aligned}&q_m|_{x=l/2}=q_m|_{x=-l/2}=0\\&\left(q'_m+\frac{1}{\mu k^2}q'''_m\right)\Big|_{x=l/2}=e^{im\psi}\left(q'_m+\frac{1}{\mu k^2}q'''_m\right)\Big|_{x-l/2}\\&\left(q''_m+\frac{f_m}{k^2}\right)\Big|_{x=l/2}=e^{im\psi}\left(q''_m+\frac{f_m}{k^2}\right)\Big|_{x=-l/2}\end{aligned}\right\} \quad (15)$$

其中

$$f_m(x)=\frac{\mu}{E'I_{\hat{w}a}}\cdot\frac{1}{\sqrt{N}}\sum_{k=1}^{N}\exp\Big[-i(k-1)m\phi\Big]m_k(x) \quad (16)$$

在(15)边界条件下，解方程(14)可得位移函数$\phi_k(x)$与相应内力$B_{wk}(x)$，$k=1,2,\cdots,N$。对$\phi_k(x)$取极限（$N\to\infty$），则可得无限跨箱形连续梁的位移函数$\phi_k^*(x)$与相应内力$B_{wk}^*(x)$，$k=1,2,\cdots N$。

2 连续梁在均布扭矩载荷作用下的扭角与双力矩

图1(a)原系统为n跨薄壁箱形连续梁，均布外扭矩M_0作用于第s跨。为获得等价的旋转周期系统，将结构、载荷拓展为$2n$跨周期结构，且$-M_0$作用于第$2n-s+1$跨见图1(b)。考虑s为中间跨，即$n=2s-1$，$N=2n=(2s-1)$，荷载函数为

$$m_s(x)=M_0,\ m_{N-s+1}(x)=-M_0;\ m_j(x)=0\quad j\neq s,\ N-s+1 \tag{17}$$

将(17)代入(16)并注意$\phi=\dfrac{2\pi}{N}$得

$$f_m(x)=\begin{cases}\dfrac{\mu}{E'I_{\hat{w}a}}\cdot\dfrac{2M_0}{\sqrt{N}}\exp[-i(s-1)m\phi] & (m=1,3,\cdots,N-1)\\ 0 & (m=2,4,\cdots,N)\end{cases} \tag{18}$$

(18)代入方程(14)得

$$d^4q_m/dx^4-\lambda^2d^2q_m/dx^2=\frac{\mu}{E'I_{\hat{w}a}}\cdot\frac{2M_0}{\sqrt{N}}\exp[-i(s-1)m\phi] \tag{19}$$

$$\left(-\frac{l}{2}<x<\frac{l}{2},\ m=1,3,\cdots,N\text{-}1\right)$$

方程(19)在(15)边界条件下的解为

$$q_m(x)=\begin{cases}(C_{m0}+C_{m1}x+C_{m2}\,\mathrm{sh}\lambda x+C_{m3}\,\mathrm{ch}\lambda x-x^2)(f_m/2\lambda^2) & (m=1,3,\cdots,N\text{-}1)\\ 0 & (m=2,4,\cdots,N)\end{cases} \tag{20}$$

其中，

$$\left.\begin{aligned}
C_{m0}&=\left(\frac{l}{2}\right)^2-\frac{c+d-(c-d)\cos m\phi}{e-f\cos m\phi}\mathrm{ch}\frac{\lambda l}{2}\\
C_{m1}&=-i\frac{(a-b)\cdot\sin m\phi}{e-f\cos m\phi}\cdot\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\\
C_{m2}&=i\frac{(a-b)\cdot\sin m\phi}{e-f\cos m\phi}\\
C_{m3}&=\frac{c+d-(c-d)\cdot\cos m\phi}{e-f\cos m\phi}
\end{aligned}\right\} \tag{21}$$

式中

$$a=\lambda^2l\,\mathrm{ch}\frac{\lambda l}{2},\qquad b=\left(2-\frac{2\lambda^2}{k^2}\right)\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\cdot\mathrm{sh}\frac{\lambda l}{2},$$

$$c=\left(2-\frac{2\lambda^2}{k^2}\right)\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right],\qquad d=\lambda^2l\,\mathrm{sh}\frac{\lambda l}{2},$$

$$e=\lambda^2\,\mathrm{ch}\frac{\lambda l}{2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right]+\lambda^2\,\mathrm{sh}^2\frac{\lambda l}{2}\left(\lambda+\frac{\lambda^3}{\mu k^2}\right),$$

$$f=\lambda^2\,\mathrm{ch}\frac{\lambda l}{2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right]-\lambda^2\,\mathrm{sh}^2\frac{\lambda l}{2}\left(\lambda+\frac{\lambda^3}{\mu k^2}\right).$$

将(20)、(21)代入(11)则得各子结构的位移函数$\phi_k(x)$（$k=1,2,\cdots,N$）。由(14)、(15)的数学特征得知，$\phi_k(x)$将为实函数，故(11)应改写为

$$\phi_k=\frac{1}{\sqrt{N}}\sum_{m=1}^{N}\mathrm{Re}[\exp[i(k-1)m\phi]q_m]\qquad(k=1,2,\cdots,N) \tag{22}$$

用(20)、(21)代入(22)便得连续梁的扭转角

$$\phi_s(x)=\frac{1}{N}\frac{M_0}{GI_k}\sum_{m=1,3,\cdots}^{N-1}\left[\left(\frac{l}{2}\right)^2+\left(\mathrm{ch}\lambda x-\mathrm{ch}\frac{\lambda l}{2}\right)\frac{c+d-(c-d)\cos m\phi}{e-f\cos m\phi}-x^2\right]$$

$$\left(-\frac{l}{2}\leq x\leq\frac{l}{2}\right) \tag{23}$$

这里的$\phi_s(x)$指载荷跨（第s跨）的扭转角，其它跨（$k\neq s$）同样不难得到。

最大扭转角发生于载荷跨跨中

$$\phi_{\max}=\phi_s(0)$$

$$=\frac{1}{N}\frac{M_0}{GI_k}\sum_{m=1,3,\cdots}^{N-1}\left[\left(\frac{l}{2}\right)^2+\left(1-\mathrm{ch}\frac{\lambda l}{2}\right)\frac{c+d-(c-d)\cos m\phi}{e-f\cos m\phi}\right] \tag{24}$$

最大双力矩发生于载荷跨跨端

$$B_{w\max}=-\beta E'I_{\hat{w}a}\ \phi_s''\left(\pm\frac{l}{2}\right)-\frac{M_0}{k^2} \qquad \left(其中\beta=\frac{1}{\mu}\right)$$

$$=\frac{M_0}{N}\sum_{m=1,3,\cdots}^{N-1}\left[\frac{2}{\lambda^2}-\mathrm{ch}\frac{\lambda l}{2}\cdot\frac{c+d-(c-d)\cos m\phi}{e-f\cos m\phi}\right]-\frac{M_0}{k^2} \tag{25}$$

现考察(23)右边级数的各项总和，有如下一般形式

$$I_n=\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}f(m\phi)=\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}f\left(\frac{m\pi}{n}\right) \tag{26}$$

设 $\Delta\theta=\frac{2\pi}{n}$，式(26)则有

$$I_n=\frac{1}{2\pi}\sum_{m=1,3,\cdots}^{2n-1}f\left(\frac{m}{2}\Delta\theta\right)\Delta\theta$$

当n趋于无穷时，级数I_n的极限可表示为如下的积分

$$\lim_{n\to\infty}\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}f(m\phi)=\frac{1}{2\pi}\int_0^{2\pi}f(\theta)d\theta \qquad (0\le\theta\le2\pi) \tag{27}$$

将这关系用于(23)，便得无限跨连续梁的扭转角

$$\phi_s^*(x)=\lim_{n\to\infty}\phi_s(x)$$

$$=\frac{M_0}{2GI_k}\left\{\left(\frac{l}{2}\right)^2+\left(\mathrm{ch}\lambda x-\mathrm{ch}\frac{\lambda l}{2}\right)\left[\frac{Af-Be+B(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}\right]-x^2\right\}$$

$$-\frac{l}{2}\le x\le\frac{l}{2} \tag{28}$$

最大扭转角发生于跨中

$$\phi_{\max}^*(x)=\phi_s^*(0)$$

$$=\frac{M_0}{2GI_k}\left\{\left(\frac{l}{2}\right)^2+\left(1-\mathrm{ch}\frac{\lambda l}{2}\right)\left[\frac{Af-Be+B(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}\right]\right\} \tag{29}$$

最大双力矩发生于跨端

$$B_{w\mathrm{mas}}^*=-\beta E'I_{\hat{w}a}\ \phi_s^{*\prime\prime}\left(\pm\frac{l}{2}\right)-\frac{M_0}{k^2}$$

$$=\frac{M_0}{2}\left\{\frac{2}{\lambda^2}-\mathrm{ch}\frac{\lambda l}{2}\left[\frac{Af-Be+B(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}\right]\right\}-\frac{M_0}{k^2} \tag{30}$$

其中， $A=\left(2-\frac{2\lambda^2}{k^2}\right)\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right]+\lambda^2 l\,\mathrm{sh}\frac{\lambda l}{2}$，

$$B=\left(2-\frac{2\lambda^2}{k^2}\right)\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right]-\lambda^2 l\,\mathrm{sh}\frac{\lambda l}{2},\qquad e,\ f\text{同前。}$$

3　连续梁在集中扭矩载荷作用下的扭角与双力矩

如图2(a)所示，原系统为n跨薄壁箱形连续梁，集中外扭矩M作用于第s跨中间处。拓展后的旋转周期系统为$2n$跨周期结构；且集中扭矩$-M$作用于第$2n-s+1$跨中间处见图2(b)，荷载函数为

$$m_s(x)=M\delta(x),\ m_{N-s+1}(x)=-M\delta(x);\ m_j(x)=0\ j\neq s,\ N-s+1 \tag{31}$$

其中$\delta(x)$为Dirac delta函数。

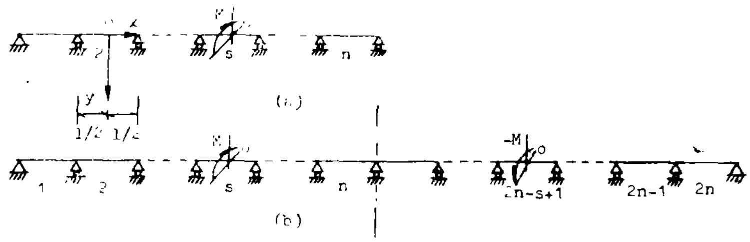

图2　作用集中扭矩薄壁箱形连续梁

Fig.2 Thin walled box girder with periodic supports to a torque M

(a) 实际体系；(b) 等效体系

将(31)代入(16)得

$$f_m(x)=\begin{cases}\dfrac{\mu}{E'I_{\hat{\omega}a}}\cdot\dfrac{2M}{\sqrt{N}}\exp[-i(s-1)m\phi]\cdot\delta(x) & (m=1,3,\cdots,N-1)\\ 0 & (m=2,4,\cdots,N)\end{cases} \tag{32}$$

(32)代入方程(14)则得

$$d^4q_m/dx^4-\lambda^2d^2q_m/dx^2=\frac{\mu}{EI_{\hat{\omega}a}}\cdot\frac{2M}{\sqrt{N}}\exp[-i(s-1)m\phi]\,\delta(x)$$
$$\left(-\frac{l}{2}<x<\frac{l}{2},\ m=1,3,\cdots,N-1\right) \tag{33}$$

方程(33)在(15)边界条件下的解为

$$q_m(x)=\begin{cases}(C_{m0}+C_{m1}x+C_{m2}\mathrm{sh}\lambda x+C_{m3}\mathrm{ch}\lambda x-x)(\bar{f}_m/2\lambda^2), & 0\leq x\leq\dfrac{l}{2}\\ (C_{m0}+C_{m1}x+C_{m2}\mathrm{sh}\lambda x+C_{m3}\mathrm{ch}\lambda x+x)(\bar{f}_m/2\lambda^2), & -\dfrac{l}{2}\leq x\leq 0\end{cases} \tag{34}$$
$$m=1,3,\cdots,N-1$$

其中，$\bar{f}_m=\dfrac{\mu}{EI_{\hat{\omega}a}}\cdot\dfrac{2M}{\sqrt{N}}\exp[-i(s-1)m\phi]$

$$\left.\begin{aligned}C_{m0}&=\frac{l}{2}+\frac{\bar{c}-\bar{d}-(\bar{c}+\bar{d})\cdot\cos m\phi}{e-f\cos m\phi}\cdot\mathrm{ch}\frac{\lambda l}{2}\\ C_{m1}&=-i\frac{(\bar{a}+\bar{b})\cdot\sin m\phi}{e-f\cos m\phi}\cdot\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\\ C_{m2}&=i\frac{(\bar{a}+\bar{b})\cdot\sin m\phi}{e-f\cos m\phi}\\ C_{m3}&=-\frac{\bar{c}-\bar{d}-(\bar{c}+\bar{d})\cos m\phi}{e-f\cos m\phi}\end{aligned}\right\} \tag{35}$$

式中，$a=\lambda^2\mathrm{ch}\frac{\lambda l}{2}$，　$\bar{b}=\frac{2\lambda^2}{k^2}\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\cdot\mathrm{sh}\frac{\lambda l}{2}$，

$$\bar{c}=\frac{2\lambda^2}{k^2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right],\qquad \bar{d}=\lambda^2\mathrm{sh}\frac{\lambda l}{2},$$

$$e=\lambda^2\mathrm{ch}\frac{\lambda l}{2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right]+\lambda^2\mathrm{sh}^2\frac{\lambda l}{2}\left(\lambda+\frac{\lambda^3}{\mu k^2}\right),$$

$$f=\lambda^2\mathrm{ch}\frac{\lambda l}{2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\mathrm{ch}\frac{\lambda l}{2}-\frac{2}{l}\mathrm{sh}\frac{\lambda l}{2}\right]-\lambda^2\mathrm{sh}^2\frac{\lambda l}{2}\left(\lambda+\frac{\lambda^3}{\mu k^2}\right).$$

将(35)、(34)代入(22)则得连续梁的扭转角

$$\phi_s(x)=\begin{cases}\frac{1}{N}\frac{M}{GI_k}\sum\limits_{m=1,3\cdots}^{N-1}\left[\frac{l}{2}+\left(\mathrm{ch}\frac{\lambda l}{2}-\mathrm{ch}\lambda x\right)\frac{\bar{c}-\bar{d}-(\bar{c}+\bar{d})\cos m\phi}{e-f\cos m\phi}-x\right] & \left(0\le x\le\frac{l}{2}\right)\\ \frac{1}{N}\frac{M}{GI_k}\sum\limits_{m=1,3\cdots}^{N-1}\left[\frac{l}{2}+\left(\mathrm{ch}\frac{\lambda l}{2}-\mathrm{ch}\lambda x\right)\frac{\bar{c}-\bar{d}-(\bar{c}+\bar{d})\cos m\phi}{e-f\cos m\phi}+x\right] & \left(-\frac{l}{2}\le x\le 0\right)\end{cases}\tag{36}$$

最大扭转角发生于载荷跨跨中

$$\phi_{\max}=\frac{1}{N}\frac{M}{GI_k}\sum_{m=1,3,\cdots}^{N-1}\left[\frac{l}{2}+\left(\mathrm{ch}\frac{\lambda l}{2}-1\right)\frac{\bar{c}-\bar{d}-(\bar{c}+\bar{d})\cos m\phi}{e-f\cos m\phi}\right]\tag{37}$$

最大双力矩发生于载荷跨跨端

$$B_{w\max}=\frac{M}{N}\sum_{m=1,3,\cdots}^{N-1}\left[\mathrm{ch}\frac{\lambda l}{2}\cdot\frac{\bar{c}-\bar{d}-(\bar{c}+\bar{d})\cos m\phi}{e-f\cos m\phi}\right]\tag{38}$$

对(36)右边级数取极限，便得无限跨连续梁的扭转角

$$\phi_s^*(x)=\lim_{n\to\infty}\phi_s(x)=\begin{cases}\frac{M}{2GI_k}\left[\frac{l}{2}+\left(\mathrm{ch}\frac{\lambda l}{2}-\mathrm{ch}\lambda x\right)\frac{\bar{A}f-\bar{B}e+\bar{B}(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}-x\right] & \left(0\le x\le\frac{l}{2}\right)\\ \frac{M}{2GI_k}\left[\frac{l}{2}+\left(\mathrm{ch}\frac{\lambda l}{2}-\mathrm{ch}\lambda x\right)\frac{\bar{A}f-\bar{B}e+\bar{B}(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}+x\right] & \left(-\frac{l}{2}\le x\le 0\right)\end{cases}\tag{39}$$

最大扭转角发生于跨中

$$\phi^*_{\max}(x)=\frac{M}{2GI_k}\left[\frac{l}{2}+\left(\mathrm{ch}\frac{\lambda l}{2}-1\right)\frac{\bar{A}f-\bar{B}e+\bar{B}(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}\right]\tag{40}$$

最大双力矩发生于跨端

$$B^{*}_{\omega\max}=\frac{M}{2}\left[\operatorname{ch}\frac{\lambda l}{2}\cdot\frac{\bar{A}f-\bar{B}e+\bar{B}(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}{f(e+f)\left(\frac{e-f}{e+f}\right)^{1/2}}\right] \tag{41}$$

其中，
$$\bar{A}=\frac{2\lambda^2}{k^2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\operatorname{ch}\frac{\lambda l}{2}-\frac{2}{l}\operatorname{sh}\frac{\lambda l}{2}\right]-\lambda^2\operatorname{sh}\frac{\lambda l}{2},$$
$$\bar{B}=\frac{2\lambda^2}{k^2}\left[\left(\lambda+\frac{\lambda^3}{\mu k^2}\right)\operatorname{ch}\frac{\lambda l}{2}-\frac{2}{l}\operatorname{sh}\frac{\lambda l}{2}\right]+\lambda^2\operatorname{sh}\frac{\lambda l}{2},\qquad e，f同前。$$

U变换法用于周期性连续系统的组合扭转分析，不仅具有未知量少，计算结果准确的优点，同时可以有效地利用结构的周期性来提高计算效率。

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Analysis of Combination Twisting of Thin-Walled Box Girders with Periodic Supports by the U-Transformation

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Abstract The combination twisting of thin-walled box girder with periodic supports has been analyzed by the U-transform method. The uncoupled governing equations with boundary conditions for single substructure can be derived using the U-transformation. Some specific loading conditions are considered. The angular displacements and bimoments have been derived.

Keywords U-transformation, combination twisting, bimoment

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