第33卷 第3期
1994年7月

中山大学学报(自然科学版)
ACTA SCIENTIARUM NATURALIUM
UNIVERSITATIS SUNYATSENI

Vol. 33 No. 3
Jul. 1994

A Unitary Calculation of $\rho^0 \to \pi^+\pi^-$ Decay and the Imaginary Part of ρ^0 Polarization Insertion

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Abstract A unitary calculation of the $\rho^0 \to \pi^+\pi^-$ decay and its relation with the ρ^0 polarization insertion are shown. The imaginary part of the ρ^0 polarization insertion is calculated by using dimensional integration method. The approaches used in this paper to draw the imaginary part of a polarization insertion are convenient and universal, and can be extensively used for other processes.

Keywords $\rho^0 \to \pi^+\pi^-$ decay, optical theorem, imaginary part of polarization insertion

The imaginary part of a propagator represents the width, or the structure of that particle, it also means the existence of the different decay channels of that particle. It is generally momentum dependent and has thresholds. When the momentum above threshold (s) the corresponding channel (s) opens. For two particle scattering, that means its phase shift will have a nonvanishing imaginary part. This process is related to the propagators of the intermediate states. So the imaginary part of the propagators is important for the scattering as well as decay processes.

There are some ways to calculate the imaginary part of the propagators, that is the imaginary part of the polarization insertion[1~3]. In this paper, however, as an example, we will present a more sophisticated, alternative approach to derive the imaginary part directly from the ρ^0 polarization insertion, instead of calculating the $\rho^0 \to \pi^+\pi^-$ decay rate by Feynman diagrams. We will show the connection between the decay rate and the imaginary part of polarization insertion.

In the second section, a unitary calculation of the $\rho^0 \to \pi^+\pi^-$ decay rate, especially the relation between the decay rate and the imaginary part of ρ^0 polarization insertion is

Received: 1993-02-24

given. In the third section, the ρ^0 polarization insertion is calculated by dimensional integration method and from which the imaginary part can be drawn. A summary and conclusions are given in the last section.

1 Unitarity and $\rho^0 \to \pi^+\pi^-$ decay

$\rho^0 \to \pi^+\pi^-$ decay is the main decay mode of ρ^0 meson[4]. The most conventional way to calculate its deacy rate $\Gamma_{\rho\to\pi\pi}$ is by pertubative Feynman diagrams for a given interaction[5,6]. The main purpose of this section is not to repeat the result, instead by unitarity or optical theorem to derive a relation between $\Gamma_{\rho\to\pi\pi}$ and the corresponding imaginary part of ρ^0 polarization insertion $\Pi_{\mu\nu}$.

The unitarity of the S matrix gives the following identity[7]

$$2\mathrm{Im}\langle i|T|i\rangle = \sum_n |\langle n|T|i\rangle|^2 \tag{1}$$

T is the transition matrix. The transition propability for a initial state $i=\rho^0$ to n-final states, by use of eq. (1) is as follows

$$P_\rho(p) = \frac{2}{3}\mathrm{Im}\sum_{\lambda=1}^{3}\langle\rho\lambda(p)|T|\rho\lambda(p)\rangle \tag{2}$$

here λ is the polarizatrion index for ρ^0. $|\rho\lambda(p)\rangle = a^+_{p\lambda}|0\rangle$. We follow the conventions of Bjorken and Drell (Relativistic Quantum Mechanics). Eq. (2) is just the optical theorem fot ρ^0 decay.

The $\rho\pi\pi$ interaction Lagrangian L_I is as follows

$$L_I = i\,\vec{j}^\mu\cdot\vec{\rho}_\mu \tag{3}$$

$$\vec{j}^\mu = i g_{\rho\pi\pi}\vec{\pi}\times\partial^\mu\vec{\pi} \tag{4}$$

since we are considering $\rho^0\to\pi^+\pi^-$ decay, so only the third component j^3_μ ($\equiv j_\mu$), ρ^3_μ ($\equiv \rho_\mu$) will contribute to the decay. Substitute the Lagrangian L_I into the T matrix and we find that

$$P_\rho(p) = -\frac{2}{3}\mathrm{Im}\frac{1}{2\omega_p V}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_\rho^2}\right)\int \mathrm{d}x\mathrm{d}y e^{ip(x-y)}\Pi_{\mu\nu}(x-y) \tag{5}$$

where $\omega_\rho = \sqrt{\vec{p}^2+m_\rho^2}$, V is the assumed renormal iztation volumn of space. $\Pi_{\mu\nu}$ is the polariation insertion of ρ^0 meson, and defined as follow

$$-i\Pi_{\mu\nu}(x-y) = \langle 0|T[j_\mu(x)j_\nu(y)]|0\rangle \tag{6}$$

So the $\rho^0\to\pi^+\pi^-$ decay rate $\Gamma_{\rho\to 2\pi}$, or the transition propability per unit time, by substituting eq. (6) in to eq. (5), can be written as follow

$$\Gamma_{\rho\to 2\pi} = \frac{1}{3\omega_\rho}\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{m_\rho^2}\right)\mathrm{Im}\Pi_{\mu\nu}(p) \tag{7}$$

$$\Pi_{\mu\nu}(p) = i g^2_{\rho\pi\pi}\int[\mathrm{d}q/(2\pi)^4](p-2a)_\mu(p-2q)_\nu\Delta_\pi(q)\Delta_\pi(p-q) \tag{8}$$

Δ_π is the pion propagator. It is easy to verify that

$$\Pi^\mu_\mu(p) = -g^2_{\rho\pi\pi}(p^2-4m_\pi^2)F_-(p^2, m_\pi^2, m_\pi^2) + \textit{real divergent term} \tag{9}$$

where function F_2 is defined in the Appendix. By using the formulae there, we have

$$\mathrm{Im}\Pi^{\mu}_{\mu}(s) = g^2_{\rho\pi\pi}(\bar{Q}^3/2\pi\sqrt{s})\,\theta(s-4m_\pi^2) \tag{10}$$

the definition of s and $\bar{Q}$ are given in the Appendix.

One can prove in a similar way that

$$\mathrm{Im}p^\mu p^\nu\Pi_{\mu\nu}(p) = 0 \tag{11}$$

which means $\mathrm{Im}\Pi_{\mu\nu}(p)$ is transverse and we will show this point explicitly in the next section.

Substitute eqs. (10, 11) into eq. (7), we find that

$$\Gamma_{\rho\to\pi\pi}(s) = g^2_{\rho\pi\pi}(\bar{Q}^3/6\pi w_\rho\sqrt{s})\,\theta(s-4m_\pi^2) \tag{12}$$

which is exactly the same as that calculated by the usual pertubative Feynman diagram method[3,5]. As stated in the introduction, the aims of this section is not only to reproduce the well known result, that is eq. (12), we emphasize on the method used here to derive eq. (12) and the relation between $\Gamma_{\rho\to2\pi}$ and $\Pi_{\mu\nu}$. We will further illustrate transversality of $\mathrm{Im}\Pi_{\mu\nu}$ in the next section.

2 Imaginary part of ρ^0 polarization insertion

In last section, a relation between $\Gamma_{\rho\to2\pi}$ and $\mathrm{Im}\Pi_{\mu\nu}$ is derived. In this section, we will calculated $\mathrm{Im}\Pi_{\mu\nu}$ by the methods given in the Appendix. The transversality of $\mathrm{Im}\Pi_{\mu\nu}$, that is eq. (11) is demonstrated and the connection between $\mathrm{Im}\Pi_{\mu\nu}$ and $\Gamma_{\rho\to2\pi}$ is shown.

The definition of $\Pi_{\mu\nu}$ is given in the last section, by using the identity

$$1/ab = \int_0^1 \mathrm{d}x/[ax + b(1-x)]^2 \tag{13}$$

eq. (8) can be written as follow

$$\Pi_{\mu\nu}(p) = ig^2_{\rho\pi\pi}\left\{p_\mu p_\nu\int_0^1 \mathrm{d}x(2x-1)^2\int\frac{\mathrm{d}q}{(2\pi)^4}\frac{1}{[q^2-p^2(x^2-x)-m_\pi^2]^2} + 4\int_0^1\mathrm{d}x\int\frac{\mathrm{d}q}{(2\pi)^4}\frac{q_\mu q_\nu}{[q^2-p^2(x^2-x)-m_\pi^2]^2}\right\} \tag{14}$$

By the dimensional integration method[9] and neglect all real terms, we find

$$\mathrm{Im}\Pi_{\mu\nu}(p) = g^2_{\rho\pi\pi}\frac{\mathrm{Im}}{16\pi^2}\int_0^1\mathrm{d}x\left\{p_\mu p_\nu(2x-1)^2 + 2g_{\mu\nu}p^2[(x-\frac{1}{2})^2-\delta^2]\right\} \cdot \ln[(x-\frac{1}{2})^2-\delta^2]\theta(p^2-4m_\pi^2) \tag{15}$$

where δ is defined in the Appendix. Eq. (15) can be further reduced in a similar way as that given in the Appendix. Finally, it can be written as

$$\begin{aligned}\mathrm{Im}\Pi_{\mu\nu}(p) &= g^2_{\rho\pi\pi}\frac{p^2\delta^3}{6\pi}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\theta(p^2-4m_\pi^2)\\ &= -\omega_\rho\Gamma_{\rho\to2\pi}(s)\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right)\end{aligned} \tag{16}$$

in the last line, eq. (12) has been substituted into the first line of eq. (16). So we can see that $\mathrm{Im}\Pi_{\mu\nu}$ is proportional to $\Gamma_{\rho\to2\pi}$ and the transversality of $\mathrm{Im}\Pi_{\mu\nu}$, that is

$$p^{\mu}\mathrm{Im}\Pi_{\mu\nu}=p^{\nu}\mathrm{Im}\Pi_{\mu\nu}=0 \qquad (17)$$

is satisfied.

3 Summary and conclusion

By the unitarity of the S matrix, we find a relation between $\Gamma_{\rho\to2\pi}$ and the imaginary past of ρ^0 polariation insertion. The decay rate calculated in this way is the same as that calculated by the usual pertubative method. But the approach used here is easy and can be easily applied to other processes. The most interesting point of this paper is that we demonstrated a new method to draw the imaginary part from a Feynman diagram and applied it to the calculation of $\mathrm{Im}\Pi_{\mu\nu}$, and verified that our method gives the result that is consistant with that of calculated by Cutkosky rule. Some of the results, eg. (16) can also be thought of as a proving for the validity of other formulae used in the literatures[3].

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Appendix

In this appendix, we will derive the imaginary part of function F_2, by using dimensional integration method.

$\mathrm{Im}F_2$ has been calculated in ref. [8] by means of Cutkosky rule, which we show here briefly.

$$F_2(p^2,m_1^2,m_2^2)=\frac{1}{(2\pi)^4}\int\mathrm{d}q\,\frac{1}{[(p+q)^2-m_1^2+i\varepsilon][q^2-m_2^2+i\varepsilon]} \qquad (A1)$$

by means of Cutkosky rule

$$\mathrm{Im}F_2(p^2,m_1^2,m_2^2)=\frac{1}{2(2\pi)^4}\int dq(2\pi i)^2\delta^+((p+q)^2-m_1^2)\delta^+(q^2-m_2^2) \quad (A2)$$

For simplicity, choose $\vec{p}=0$, and we find

$$\mathrm{Im}F_2=-\frac{\overline{Q}}{8\pi p_0}\theta(p_0^2-(m_1+m_2)^2) \quad (A3)$$

$$\overline{Q}^2=\frac{1}{4p_0^2}(p_0^2-(m_1+m_2)^2)(p_0^2-(m_1-m_2)^2) \quad (A4)$$

the Lorentz in variance of F_2 ensure that p_0^2 can be replaced by $p^2=s$. In case of $m_1=m_2=m_\pi$,

$$\mathrm{Im}F_2=-\frac{\overline{Q}}{8\pi\sqrt{s}}\theta(s-4m_\pi^2) \quad (A5)$$

$$\overline{Q}=\frac{1}{2}\sqrt{s-4m_\pi^2} \quad (A6)$$

Next we will show our method to derive $\mathrm{Im}F_2$. In ref. (9), we have shown the calculation of F_2 by dimensional integration, the result is

$$F_2(p^2,m_\pi^2,m_\pi^2)=-\frac{\Gamma(\varepsilon)}{16\pi^2}+\frac{1}{16\pi^2}\int_0^1 dx\ln[p^2(x^2-x)+m_\pi^2]+o(\varepsilon) \quad (A7)$$

here $\varepsilon\to 0$, and we have chosen $m_1=m_2=m_\pi$. The imaginary part appears only when the argument in the logarithm becomes negative, that is

$$x-\frac{1}{2}-\delta<0, \text{ and } x-\frac{1}{2}+\delta>0 \quad (A8)$$

it is true only when $p^2>4m_\pi^2$. $\delta=\sqrt{(\frac{1}{4}-\frac{m_\pi^2}{p^2})}=\frac{\overline{Q}}{\sqrt{s}}$. For other case $\mathrm{Im}F_2=0$. So

$$\mathrm{Im}F_2=\frac{1}{16\pi^2}\mathrm{Im}\int_{\frac{1}{2}-\delta}^{\frac{1}{2}+\delta}dx\ln[(x-\frac{1}{2})^2-\delta^2]=\frac{2\pi\delta}{16\pi^2} \quad (A9)$$

this is the same equation as (A5).

$\rho^0\to\pi^+\pi^-$衰变的么正计算和ρ^0极化插入的虚部

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摘　要　本文给出了$\rho^0\to\pi^+\pi^-$衰变的么正计算及其衰变率和ρ^0极化插入的关系. ρ^0极化插入的虚部是用维度积分的方法来计算. 极化插入虚部的计算方法是普适的.

关键词　$\rho^0\to\pi^+\pi^-$衰变, 光学定理, 极化插入虚部

分类号　O412.3

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