第 33 卷 第 3 期
1994 年 7 月

中山大学学报(自然科学版)
ACTA SCIENTIARUM NATURALIUM UNIVERSITATIS SUNYATSENI

Vol. 33 No. 3
Jul. 1994

具有两个广义位移连续梁静力分析的精确解*

陈森炎 蔡承武
(中山大学应用力学与工程系,广州 510275)

摘 要 应用U变换,精确地分析了周期性连续深梁的静力问题.结合两种典型荷载条件,给出了相应的位移和内力的解析表达式,包括跨数趋于无穷时的极限情形.

关键词 具有两个广义位移梁,U变换,精确解
分类号 O342

关于具有一个广义位移的周期性连续梁,应用 U 变换研究其静力问题[1]与动力问题[2]均已获得解析解.众所周知,具有一个广义位移梁的适用范围有一定的局限性,只适用于浅梁情况,随着工程技术的发展而深梁(指跨高比 $l/h \leqslant 2.5$ 的梁)的问题将会遇到愈来愈多.本文基于 Timoshenko 提出具有两个广义位移梁的理论[3],将 U 变换法应用于周期性连续深梁的弯曲分析中,获得了其它方法难以得到的任意跨连续深梁的精确解,其解也适用于周期性连续夹层梁和层梁.

1 控制方程

考察一具有弯曲刚度 D 与剪切刚度 C 的无限长深梁,支承在周期性分布的刚性支座上(如图 1),相邻两支座间的跨长为 l,子结构编号为 $1,2,3\cdots N$,众所周知,在无限长连续深梁的有限范围内受到任意载荷作用时,均不会在梁的两个无穷远端点处引起位移与内力,因此可假想将两个无穷远端点“连接”起来,将无限长连续深梁视为具有相同子结构,当子结构数目 N 趋于无穷时的迴转周期结构,于子结构上建立局部坐标系(见图 1),并设原点位于跨度中点,在局部坐标系中子结构的平衡方程[3]为

$$\left.\begin{aligned} &-\frac{\mathrm{d}}{\mathrm{d}x}\left[C\left(\frac{\mathrm{d}W_k}{\mathrm{d}x}-\theta_k\right)\right]=g_k(x) \\ &-\frac{\mathrm{d}}{\mathrm{d}x}\left(D\frac{\mathrm{d}\theta_k}{\mathrm{d}x}\right)-C\left(\frac{\mathrm{d}W_k}{\mathrm{d}x}-\theta_k\right)=m_k(x) \\ &\qquad -\frac{l}{2}<x<\frac{l}{2},\ k=1,2\cdots,N \end{aligned}\right\} \tag{1}$$

收稿日期:1992-11-27
* 国家自然科学基金资助项目

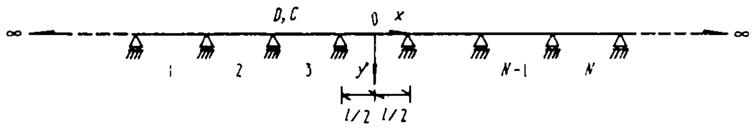

图1 周期性连续深梁

Fig. 1 Continuous deep beam with periodic Supports

其中，$W_k(x)$，$\theta_k(x)$分别表示第K个子结构的挠度函数和转角函数；$g_k(x)$，$m_k(x)$分别表示横向分布荷载函数和分布力矩函数.

方程组(1)应满足约束条件与相邻子结构间的连续条件.即

$$\left.\begin{aligned} &W_k(l/2) = W_k(-l/2) = 0 \\ &\theta_k(l/2) = \theta_{k+1}(-l/2) \\ &\theta'_k(l/2) = \theta'_{k+1}(-l/2) \qquad k = 1,2,\cdots,N \end{aligned}\right\} \tag{2}$$

其中，$\theta_{N+1} \equiv \theta_1$，最后一式代表弯矩连续.

对微分方程组(1)和式(2)作U变换[4]，令

$$W_k(x) = \frac{1}{\sqrt{N}}\sum_{m=1}^{N}\exp[i(k-1)m\psi]q_m(x) \tag{3}$$

$$\theta_k(x) = \frac{1}{\sqrt{N}}\sum_{m=1}^{N}\exp[i(k-1)m\psi]Q_m(x) \tag{4}$$

其中，$\psi = 2\pi/N$，$i = \sqrt{-1}$.

方程组(1)和式(2)则变为

$$\left.\begin{aligned} &-\frac{d}{dx}[C(\frac{dq_m}{dx} - Q_m)] = f_m(x) \\ &-\frac{d}{dx}(D\frac{dQ_m}{dx}) - C(\frac{dq_m}{dx} - Q_m) = F_m(x) \\ &\qquad\qquad -\frac{l}{2} < x < \frac{l}{2}, m = 1,2,\cdots,N \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} &q_m(l/2) = q_m(-l/2) = 0 \\ &Q_m(l/2) = \exp(\mathrm{i}m\psi)Q_m(-l/2) \\ &Q'_m(l/2) = \exp(\mathrm{i}m\psi)Q'_m(-l/2) \end{aligned}\right\} \tag{6}$$

其中，q_m，Q_m分别为第m个广义位移和广义转角；而$f_m(x)$，$F_m(x)$为其对应的广义荷载.即

$$\left.\begin{aligned} &f_m(x) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}\exp[-i(k-1)m\psi]g_k(x) \\ &F_m(x) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}\exp[-i(k-1)m\psi]m_k(x) \end{aligned}\right\} \tag{7}$$

在边界条件(6)下，解方程组(5)可得$q_m(x)$和$Q_m(x)$，将$q_m(x)$和$Q_m(x)$分别代入式(3)和(4)便可得到$W_k(x)$和$\theta_k(x)$，当N趋于无穷时，$W_k(x)$和$\theta_k(x)$的极限[2]为：

$$W_k^*(x)=\lim_{N\to\infty}W_k(x)=\frac{1}{2\pi}\int_0^{2\pi}Re\{\exp[i(k-1)\varphi]q(\varphi,x)\}\mathrm{d}\varphi \tag{8}$$

$$\theta_k^*(x)=\lim_{N\to\infty}\theta_k(x)=\frac{1}{2\pi}\int_0^{2\pi}Re\{\exp[i(k-1)\varphi]Q(\varphi,x)\}\mathrm{d}\varphi \tag{9}$$

其中 $q(\varphi,x)$、$Q(\varphi,x)$ 的定义为：

$$q(m\psi,x)\equiv\sqrt{N}q_m(x),\quad Q(m\psi,x)\equiv\sqrt{N}Q_m(x)$$

2　连续深梁某一跨在均布载荷 P_0 作用下的精确解

如图 2(a) 所示原结构为 n 跨连续深梁，均布载荷 P_0 作用于第 s 跨，为获得与之等价的迴转周期结构，设想将结构对称，载荷反对称拓展为 $N=2n$ 跨周期结构，且载荷 $-P_0$ 作用于第 $N-s+1$ 跨，见图 2(b)。

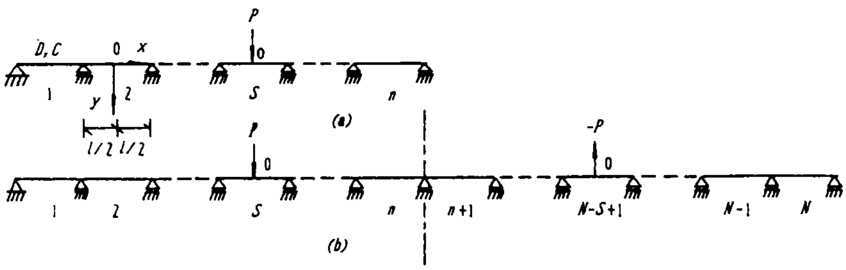

图 2　作用有均布载荷 P_0 的连续深梁

Fig. 2　Continuous deep beam with periodic supports subjected to the uniform load P_0

(a) 实际体系　　(b) 等效体系

考虑载荷作用在中间跨的情形，即 $n=2s-1$，载荷函数为

$$g_s(x)=P_0,\quad g_{N-s+1}(x)=-P_0;\quad g_j(x)=0,j\neq S,N-s+1;$$
$$m_j(x)=0,j=1,2,\cdots,N \tag{10}$$

将(10) 代入(7) 得广义载荷

$$f_m=\begin{cases}(2p_0/\sqrt{N})\exp[-i(S-1)m\psi] & m=1,3,\cdots,N-1\\ 0 & m=2,4,\cdots,N\end{cases}$$
$$F_m=0\qquad m=1,2,\cdots,N \tag{11}$$

式(11) 代入方程组(5)，结合边界条件(6) 其解可表为

$$\left.\begin{aligned}&q_m(x)=\frac{2P_0}{D\sqrt{N}}\exp[-i(s-1)m\psi](C_{m0}+C_{m1}x+C_{m2}x^2+C_{m3}x^3+\frac{x^4}{24})\\&Q_m(x)=\frac{2P_0}{D\sqrt{N}}\exp[-i(s-1)m\psi](D_{m0}+D_{m1}x+D_{m2}x^2+\frac{x^3}{6})\\&\qquad\qquad m=1,3,\cdots,N-1\\&q_m(x)=Q_m(x)=0\qquad m=2,4,\cdots,N\end{aligned}\right\} \tag{12}$$

式中，

$$C_{m_0}=\frac{l^4}{384}\left(\frac{a+b\cos m\psi}{e+f\cos m\psi}\right);\qquad C_{m_1}=-i\frac{l^3}{96}\left(\frac{\sin m\psi}{e+f\cos m\psi}\right);$$

$$C_{m_2}=-\frac{l^2}{16}\left(\frac{c+d\cos m\psi}{e+f\cos m\psi}\right);\quad C_{m3}=-\frac{4}{l^2}c_{m_1}.$$

$$D_{m_0}=\left(1-24\frac{D}{Cl^2}\right)C_{m1};\qquad D_{m_1}=2C_{m_2}+\frac{D}{C};\qquad D_{m_2}=-\frac{12}{l^2}C_{m_1}.$$

其中，$a=126\left(\frac{D}{Cl^2}\right)+288\left(\frac{D}{Cl^2}\right)^2+4$，　$b=18\left(\frac{D}{Cl^2}\right)-288\left(\frac{D}{Cl^2}\right)^2-1$，

$c=22\left(\frac{D}{Cl^2}\right)+48\left(\frac{D}{Cl^2}\right)^2+1$，　$d=2\left(\frac{D}{Cl^2}\right)-48\left(\frac{D}{Cl^2}\right)^2$，

$e=2+6\left(\frac{D}{Cl^2}\right)$，　$f=1-6\left(\frac{D}{Cl^2}\right)$.

将 $q_m(x)$ 和 $Q_m(x)$ 代入对应的(3)和(4)便得各子结构的挠度和转角函数.

$$W_s(x)=\frac{P_0l^4}{384D}\cdot\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}\left[\frac{a+b\cos m\psi}{e+f\cos m\psi}-24\left(\frac{c+d\cos m\psi}{e+f\cos m\psi}\right)\left(\frac{x}{l}\right)^2+16\left(\frac{x}{l}\right)^4\right]$$
$$-\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\qquad(13)$$

$$\theta_s(x)=\frac{P_0l^3}{48D}\cdot\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}\left\{\left[48\frac{D}{Cl^2}-6\left(\frac{c+d\cos m\psi}{e+f\cos m\psi}\right)\right]\left(\frac{x}{l}\right)+8\left(\frac{x}{l}\right)^3\right\}$$
$$-\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\qquad(14)$$

对应的弯矩和剪力为

$$M_s(x)=-D\frac{d\theta_s(x)}{dx}=\frac{P_0l^2}{8}\cdot\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}\left[\frac{c+d\cos m\psi}{e+f\cos m\psi}-8\frac{D}{Cl^2}-4\left(\frac{x}{l}\right)^2\right]\qquad(15)$$

$$V_s(x)=C\left[\frac{dW_s(x)}{dx}-\theta_s(x)\right]=\frac{P_0}{n}\sum_{m=1,3,\cdots}^{2n-1}(-x)=-P_0x\qquad(16)$$

将 $q(m\psi,x)\equiv\sqrt{N}q_m(x)$，$Q(m\psi,x)\equiv\sqrt{N}Q_m(x)$ 分别代入积分表达式(8)和(9)，经积分后可得无限长连续深梁各子结构的挠度函数和转角函数.

$$W_s^*(x)=\lim_{n\to\infty}W_s(x)$$
$$=\frac{P_0l^4}{384D}\left\{\frac{af-be+b\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}-24\left[\frac{cf-de+d\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}\right]\left(\frac{x}{l}\right)^2+16\left(\frac{x}{l}\right)^4\right\}$$
$$-\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\qquad(17)$$

$$\theta_s^*(x)=\lim_{n\to\infty}\theta_s(x)$$
$$=\frac{P_0l^3}{48D}\left\{\left[48\frac{D}{cl^2}-6\frac{cf-de+d\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}\right]\left(\frac{x}{l}\right)+8\left(\frac{x}{l}\right)^3\right\},\ -\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\qquad(18)$$

无限长连续深梁载荷跨的弯矩和剪力为

$$M_s^*(x)=-D\frac{d\theta_s^*(x)}{dx}=\frac{P_0l^2}{8}\left[\frac{cf-de+d\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}-8\frac{D}{cl^2}-4\left(\frac{x}{l}\right)^2\right]\qquad(19)$$

$$V_s^*(x)=C\left[\frac{dW_s^*(x)}{dx}-\theta_s^*(x)\right]=-P_0x\qquad(20)$$

最大挠度、弯矩和最大转角、剪力将分别发生于有限、无限长连续深梁载荷跨的跨中

点处和跨端处.

3　连续深梁某一跨跨中在集中载荷 P 作用下的精确解

如图 3(a) 所示原结构为 n 跨连续深梁.集中载荷 P 作用于第 S 跨中点处(设 s 位于中间跨).拓展后的回转周期结构为 $N = 2n$ 跨周期结构.且载荷 $-P$ 作用于第 $N-s+1$ 跨中点处.见图 3(b).

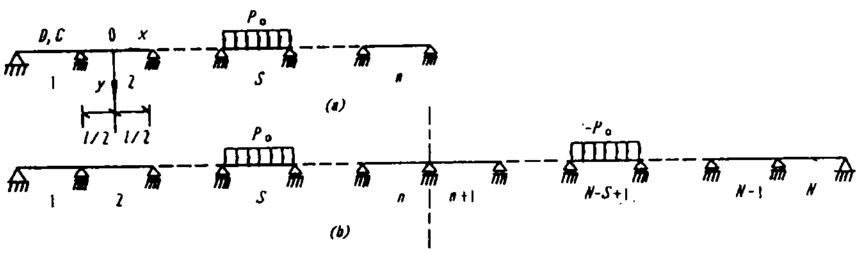

图 3　作用有集中载荷 P 的连续深梁

Fig. 3　Continuous deep beam with periodic Supports subjected to a concentrated load P

(a) 实际体系　　(b) 等效体系

载荷函数为

$$g_s(x) = P\delta(x),\ g_{N-s+1}(x) = -P\delta(x);\ g_j(x) = 0, j \neq s, N-s+1;$$
$$m_j(x) = 0, j = 1,2\cdots,N \qquad (21)$$

其中 $\delta(x)$ 为 Dirac delta 函数.将式(21) 代入式(7) 得广义载荷

$$f_m(x) = \begin{cases} \dfrac{2P}{\sqrt{N}}\exp[-i(s-1)m\psi]\delta(x) & m = 1,3,\cdots,N-1 \\ 0 & m = 2,4,\cdots,N \end{cases} \qquad (22)$$
$$F_m(x) = 0 \qquad m = 1,2,\cdots,N$$

式(22) 代入方程组(5),结合边界条件(6),其解可表为

$$\left.\begin{aligned} q_m(x) &= \frac{2P}{D\sqrt{N}}\exp[-i(s-1)m\psi](C_{m0} + C_{m1}x + C_{m2}x^2 + C_{m3}x^3 - \frac{D}{2C}|x| + \frac{|x|^3}{12}) \\ &\qquad (-l/2) \leqslant x \leqslant (l/2) \\ Q_m(x) &= \frac{2P}{D\sqrt{N}}\exp[-i(s-1)m\psi](D_{m0} + D_{m1}x + D_{m2}X^2 + \mathrm{sgn}x \cdot \frac{x^2}{4}) \\ &\qquad (-l/2) \leqslant x \leqslant (l/2) \\ &\qquad m = 1,3,\cdots,N-1 \\ q_m(x) &= Q_m(x) = 0 \qquad m = 2,4,\cdots,N \end{aligned}\right\} \qquad (23)$$

式中,　$C_{m0} = \dfrac{l^3}{384}(\dfrac{\bar{a} - \bar{b}\cos m\psi}{e + f\cos m\psi} + 96\dfrac{D}{cl^2})$;　　$C_{m1} = -i\dfrac{l^2}{64}(\dfrac{\sin m\psi}{e + f\cos m\psi})$;

$$C_{m2}=-\frac{l}{32}\left(\frac{\bar{c}+\bar{d}\cos m\psi}{e+f\cos m\psi}\right);\qquad C_{m3}=-\frac{4}{l^2}C_m;$$

$$D_{m0}=\left(1-24\frac{D}{cl^2}\right)C_{m1};\qquad D_{m1}=2C_{m2};\qquad D_{m2}=-\frac{12}{l^2}C_{m1}.$$

其中，$\bar{a}=7+48\left(\frac{D}{cl^2}\right)$，$\bar{b}=1+48\left(\frac{D}{cl^2}\right)$，$\bar{c}=5+24\left(\frac{D}{cl^2}\right)$，

$\bar{d}=1-24\left(\frac{D}{cl^2}\right)$，$e=2+6\left(\frac{D}{cl^2}\right)$，$f=1-6\left(\frac{D}{cl^2}\right)$.

将 $q_m(x)$，$Q_m(x)$ 分别代入对应的式(3)，(4)则得各子结构挠度和转角函数.

$$W_s(x)=\frac{pl^3}{384D}\cdot\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}\left[\left(\frac{\bar{a}-\bar{b}\cos m\psi}{e+f\cos m\psi}+96\frac{D}{cl^2}\right)-192\frac{D}{cl^2}\left(\frac{|x|}{l}\right)-12\left(\frac{\bar{c}+\bar{d}\cos m\psi}{e+f\cos m\psi}\right)\left(\frac{x}{l}\right)^2+32\left(\frac{|x|}{l}\right)^3\right].\quad -\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\tag{24}$$

$$\theta_s(x)=\frac{pl^2}{16D}\cdot\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}\left[-\left(\frac{\bar{c}+\bar{d}\cos m\psi}{e+f\cos m\psi}\right)\left(\frac{x}{l}\right)+4\,\mathrm{sgn}\,x\left(\frac{x}{l}\right)^2\right].\quad -\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\tag{25}$$

对应的弯矩和剪力为

$$M_s(x)=-D\theta'_s(x)=\frac{pl^2}{16}\cdot\frac{1}{n}\sum_{m=1,3,\cdots}^{2n-1}\left[\frac{\bar{c}+\bar{d}\cos m\psi}{e+f\cos m\psi}-8\,\mathrm{sgn}\,x\left(\frac{x}{l}\right)\right]\tag{26}$$

$$V_s(x)=C[W'_s(x)-\theta_s(x)]=\frac{p}{n}\sum_{m=1,3,\cdots}^{2n-1}\left(-\mathrm{sgn}\,x\,\frac{1}{2}\right)=-\mathrm{sgn}\,x\cdot\frac{p}{2}\tag{27}$$

同理，将 $q(m\psi,x)\equiv\sqrt{N}q_m(x)$，$Q(m\psi,x)\equiv\sqrt{N}Q_m(x)$ 分别代入式(8)和(9)，经积分后则得无限连续深梁各子结构的挠度和转角函数.

$$W_s^*(x)=\lim_{n\to\infty}W_s(x)=\frac{pl^3}{384D}\left\{\frac{\bar{a}f+\bar{b}e-\bar{b}\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}+96\left(\frac{D}{cl^2}\right)-192\frac{D}{cl^2}\left(\frac{|x|}{l}\right)-12\left[\frac{\bar{c}f-\bar{d}e+\bar{d}\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}\right]\left(\frac{x}{l}\right)^2+32\left(\frac{|x|}{l}\right)^3\right\}.\quad -\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\tag{28}$$

$$\theta_s^*(x)=\lim_{n\to\infty}\theta_s(x)=\frac{pl^2}{16D}\left\{-\left[\frac{\bar{c}f-\bar{d}e+\bar{d}\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}\right]\left(\frac{x}{l}\right)+4\,\mathrm{sgn}\,x\left(\frac{x}{l}\right)^2\right\}\quad -\frac{l}{2}\leqslant x\leqslant\frac{l}{2}\tag{29}$$

无限长连续深梁载荷跨的弯矩和剪力为

$$M_s^*(x)=-D\theta_s^{*\prime}(x)=\frac{pl}{16}\left[\frac{\bar{c}f-\bar{d}e+\bar{d}\sqrt{3(e-f)}}{f\sqrt{3(e-f)}}-8\,\mathrm{sgn}\,x\left(\frac{x}{l}\right)\right]\tag{30}$$

$$V_s^*(x)=C[W'_s(x)-\theta_s^*(x)]=\begin{cases}-p/2 & 0<x\leqslant\frac{l}{2}\\ p/2 & -\frac{l}{2}\leqslant x<0\end{cases}\tag{31}$$

最大挠度、弯矩和最大转角、剪力将分别发生于有限、无限长连续深梁的集中载荷作用点处和跨端处.

为了说明剪切变形的作用对广义位移及其内力的影响，现将上述两算例的结果，以 $\frac{D}{cl^2}$ 为参数的最大挠度、弯矩与跨数之间关系汇于表1.

由表 1 可以看到，考虑剪切变形作用，连续深梁的广义位移和内力将随无量纲参数 $\frac{D}{cl^2}$ 的增加而增大，并由此得知，对于同剖面的梁，跨度 l 愈小，剪切变形的作用就愈大，只有当 $C=\infty$，$\frac{D}{cl^2}=0$ 时，才得到文〔1〕不考虑剪切变形影响，具有一个广义位移连续梁的计算结果.

表 1　最大挠度与最大弯矩

Tab. 1　Maximum deflections and maximum bending moments

参数 D/cl^2	0					
跨数　N	1	3	5	7	9	∞
① $W_{max}\cdot(P_0l^4/384D)^{-1}$	5.00000	2.60000	2.47368	2.46479	2.46415	2.46410162
② $W_{max}\cdot(Pl^3/384D)^{-1}$	8.00000	4.40000	4.21053	4.19747	4.19622	4.19615242
① $M_{max}\cdot(P_0l^2/16)^{-1}$	2.00000	1.20000	1.15789	1.15499	1.15472	1.154700538
② $M_{max}\cdot(Pl/16)^{-1}$	4.00000	2.80000	2.73684	2.73249	2.73208	2.732050807
参数 D/cl^2	0.5					
跨数　N	1	3	5	7	9	∞
① $W_{max}\cdot(P_0l^4/384D)^{-1}$	29.00000	27.50000	27.42105	27.41772	27.41743	27.41739861
② $W_{max}\cdot(Pl^3/384D)^{-1}$	56.00000	53.75000	53.63157	53.62659	53.62615	53.62613645
① $M_{max}\cdot(P_0l^2/16)^{-1}$	2.00000	1.50000	1.47368	1.47258	1.47248	1.472466204
② $M_{max}\cdot(Pl/16)^{-1}$	4.00000	3.25000	3.21052	3.20886	3.20872	3.20871215
参数 D/cl^2	1.0					
跨数　N	1	3	5	7	9	∞
① $W_{max}\cdot(P_0l^4/384D)^{-1}$	53.00000	51.90909	51.72847	51.70540	51.70241	51.70197760
② $W_{max}\cdot(Pl^3/384D)^{-1}$	104.00000	102.36363	102.09271	102.05810	102.05361	102.0530012
① $M_{max}\cdot(P_0l^2/16)^{-1}$	2.00000	1.63636	1.57616	1.56846	1.56747	1.567343200
② $M_{max}\cdot(Pl/16)^{-1}$	4.00000	3.45455	3.36424	3.35270	3.35120	3.351000389

① 表示中间跨作用均布载荷情形；② 表示中间跨跨中作用集中载荷情形

参考文献

1　Cheung Y K, Chan H C, Cai C W. Exact method for static analysis of periodic structures. ASCE,J. Engineering Mechanics. 1989, 115(2): 451 ~ 434.

2　Cai C W, Cheung Y K, Chan H C. Dynamic Response of infinite continuous beams subjected to a moving force — an exact method. J Sound and Vibration. 1988,123(3);461 ~ 472.

3　胡海昌著. 弹性力学的变分原理及其应用. 北京：科学出版社，1981

4　蔡承武. 旋转周期结构的分析解. 中山大学学报(自然科学版)，1986(2):64 ~ 72

Exact Solution for Static Analysis of Continuous Deep Beams with Periodic Supports

Chen Senyan * *Cai Chengwu*

Abtract The static analysis of continuous deep beams with arbitrary number of periodic supports has been made exactly by the U-transform method. Two typical loading conditions — the concentrated and the uniform load acting on the middle span are Considered. The analytical solutions for transverse displacements and stress resultants have been derived.

Keywords continuous deep beam, U transformation, exact solution

* Department of Applied Mechanics and Engineering, Zhongshan University Guangzhon 510275