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带电荷黑洞的修正熵⒇

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摘　要　采用“brick wall”模型研究了二维和四维带电黑洞的修正熵，得到了和用泛函积分计算一致的结果．对于极端黑洞，二维黑洞的修正熵为零，但四维黑洞的修正熵存在更高次的发散．

关键词　带电黑洞，修正熵，“brick wall”模型
分类号　O 412.1

黑洞可以看作一个热力学系统，黑洞物理研究的一个重要方面是黑洞的熵的问题．根据热力学第一定理，黑洞的热力学熵 S 和自由能 F 有

$$\mathrm{d}F = -S\mathrm{d}T \tag{1}$$

其中，T 是系统的温度．自由能除了有经典 (树图) 贡献外，还包括量子 (单圈) 修正．因此，热力学熵也有经典部分 S_{class} 和量子修正的部分 S_q:

$$S = S_{\mathrm{class}} = S_q \tag{2}$$

其中，S_{class} 就是通常的 Bekenstein-Hawking 熵．本文采用“brick wall”模型[1]对二维和四维带电荷 dilaton 黑洞的修正熵进行了研究．近来的研究表明带电的非极端黑洞和极端洞 $(m=q)$ 有着不同的拓朴结构[2,3]．

1　二维带电荷 dilaton 黑洞的修正熵的计算

二维黑洞虽然只是一个玩具模型，但它可以得到一些精确的结果，有助于对问题的了解．考虑电磁耦合的 string-inspired 二维 dilaton 背景作用量

$$I_{gr} = \int -\mathrm{d}x\,\mathrm{d}t\,\sqrt{-g}\,\mathrm{e}^{-2H}\left[-R + 4(\nabla H)^2 + \lambda^2 - (1/4)F^2\right] \tag{3}$$

其中，$H(x)$ 是 dilaton 场．通过变分可以求得[4]: $\mathrm{d}s^2 = -g(x)\,\mathrm{d}t^2 + \frac{1}{g(x)}\mathrm{d}x^2$，$g(x) = 1 - 2m\mathrm{e}^{-\lambda x} + q^2\mathrm{e}^{-2\lambda x}$，$H(x) = -(1/2)x$，$F = \sqrt{2}q\lambda\,\mathrm{e}^{-\lambda x}$，$\lambda > 0$.

黑洞的视解为 $x_\pm = \lambda^{-1}\ln(m \pm \sqrt{m^2 - q^2})$，令 $z(x) = \mathrm{e}^{-\lambda x}$，$z_\pm = \mathrm{e}^{\lambda x_\pm} = m \pm \sqrt{m^2 - q^2}$，于是 $g(x) = (1 - z_+/z)(1 - z_-/z)$，Hawking 温度为 $T_H = U_H^{-1}$，

$$U_H = \frac{4\pi}{g'}\bigg|_{x=x_+} = \frac{2\pi(m \pm \sqrt{m^2 - q^2})}{\lambda\sqrt{m^2 - q^2}} = \frac{4\pi z_+}{\lambda(z_+ - z_-)} \tag{4}$$

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下面采用 t'Hooft的"brick wall"模型[1]计算修正熵 S_q.

考虑在黑洞背景中传播的无质量中性标量粒子,作用量为

$$I(h) = -\frac{1}{2}\int \mathrm{d}x\,\mathrm{d}t\,\sqrt{-g}\,(\nabla H)^2 \tag{5}$$

运动方程为

$$-g^{-1}\frac{\partial^2 h}{\partial t^2} + \frac{\partial}{\partial x}\left(g\frac{\partial h}{\partial x}\right) = 0 \tag{6}$$

令 $h(x,t) = U(x)\exp(ikt)$,(6)式化为

$$k^2 U + g\frac{\partial}{\partial x}\left(g\frac{\partial U}{\partial x}\right) = 0 \tag{7}$$

令 $y(x)$满足$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{g(x)}$,则(7)式化为

$$\frac{\partial^2 U}{\partial y^2} + k^2 U = 0 \tag{8}$$

利用 t'Hooft边界条件[1],当 $x \leqslant x_+ + X$时,$U(x) = 0$;当 $x \geqslant L$ 时,$U(x) = 0$. 其中,X为紫外截断,L 为红外截断. (8)式的解为

$$U(y(x)) = \sin[k(y(x) - y(x_+ + X))] \tag{9}$$

能级为

$$n\pi = k(y(x) - y(x_+ + X)) = \int_{y(x_+ + X)}^{y(L)} k\,\mathrm{d}y = \int_{z_+ e^{\lambda W}}^{e^{\lambda L}} k\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}z}\mathrm{d}z =$$

$$\int_{z_+ e^{\lambda W}}^{e^{\lambda L}} \frac{k}{\lambda g(z) z}\mathrm{d}z = \frac{k}{\lambda(z_+ - z_-)}\left[z_+ \lg\frac{e^{\lambda L} - z_+}{z_+ e^{\lambda W} - z_+} - z_- \lg\frac{e^{\lambda L} - z_-}{z_+ e^{\lambda W} - z_-}\right] \tag{10}$$

因 $UF = \sum_n \ln(1 - e^{-Uk(n)})$,定义态密度为 $g(k) = \dfrac{\mathrm{d}n(k)}{\mathrm{d}k}$,$\sum_n \to \int \mathrm{d}k\,g(k)$,则

$$F = -\int_0^\infty \mathrm{d}k\,\frac{n(k)}{e^{Uk} - 1} \tag{11}$$

将(10)式代入(11)式得

$$F = -\frac{1}{\pi\lambda(z_+ - z_-)}\left[z_+ \lg\frac{e^{\lambda L} - z_+}{z_+ e^{\lambda W} - z_+} - z_- \lg\frac{e^{\lambda L} - z_-}{z_+ e^{\lambda W} - z_-}\right]\int_0^\infty \mathrm{d}k\,\frac{k}{e^{Uk} - 1} =$$

$$-\frac{\pi}{6\lambda U^2(z_+ - z_-)}\left[z_+ \lg\frac{e^{\lambda L} - z_+}{z_+ e^{\lambda W} - z_+} - z_- \lg\frac{e^{\lambda L} - z_-}{z_+ e^{\lambda W} - z_-}\right] \tag{12}$$

由 $S = U^2 \partial_U F|_{U = U_H}$,得

$$S_q = \frac{1}{24 z_+}\left[z_+ \lg\frac{e^{\lambda L} - z_+}{z_+ e^{\lambda W} - z_+} - z_- \lg\frac{e^{\lambda L} - z_-}{z_+ e^{\lambda W} - z_-}\right] \tag{13}$$

对于极端黑洞$(m = q)$,$z_+ = z_-$,所以 $S_q^{ext} = 0$.

当 $q = 0$,即 $z_- = 0$,因 X很小,所以 $S_q = \dfrac{1}{12}\lg\dfrac{e^{\lambda L} - z_+}{\lambda z_+ X}$. 因为 $L \gg x_+$,用 $L + x_+$ 代替 L,上式化为

$$S_q = \frac{1}{12}\lg\frac{e^{\lambda L} - 1}{\lambda X} \tag{14}$$

这就是文献[5]的结果.

2 四维 Reissuer-Nordstrom黑洞的修正熵的计算

下面沿用文献[6]的方法,但并不采用其中的 Pauli-Villars正规法,仍继续采用"brick wall”模型．R-N黑洞的度规为

$$ds^2 = -g\,dt^2 + g^{-1}dr^2 + r^2 d\Omega^2,\quad g = 1 - 2m/r + q^2/r^2 = (1 - r_+/r)(1 - r_-/r),$$
$$r_\pm = m \pm \sqrt{m^2 - q^2} \tag{15}$$

在黑洞背景中传播的无质量中性标量粒子的运动方程为

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\,g^{\mu\nu}\frac{\partial\psi}{\partial x^\nu}\right) = 0 \tag{16}$$

令 $\psi = e^{i\omega t}Y_{lm}(\theta,\varphi)f(r)$，代入(16)式，得径向方程为

$$\frac{r^2\omega^2}{(r-r_+)(r-r_-)}f(r) + \frac{1}{r^2}\partial_r[(r-r_+)(r-r_-)\partial_r f(r)] - \frac{l(l+1)}{r^2}f(r) = 0 \tag{17}$$

采用 WKB 近似，$f(r) = e^{iS(r)}$，得

$$\left(\frac{\partial S}{\partial r}\right)^2 = g^{-2}\left[\omega^2 - g\frac{l(l+1)}{r^2}\right],\quad S = \int g^{-1}\left[\omega^2 - g\frac{l(l+1)}{r^2}\right]^{1/2}dr \tag{18}$$

采用 t'Hooft 边界条件[1]，$\psi(r_+ + \varepsilon) = \psi(L) = 0$，能级为

$$n\pi = \int_{r_+ + \varepsilon}^{L} g^{-1}\left[\omega^2 - g\frac{l(l+1)}{r^2}\right]^{1/2}dr \tag{19}$$

由于角动量简并，总波数近似为

$$N\pi = \int(2l+1)n\pi\,dl = \int_{r_+ + \varepsilon}^{L}dr\,g^{-1}\int_0^{l}(2l+1)\left[\omega^2 - g\frac{l(l+1)}{r^2}\right]^{1/2} =$$
$$\frac{2}{3}\int_{r_+ + \varepsilon}^{L} g^{-2}r^2\omega^3 dr = \frac{2}{3}\int_{r_+ + \varepsilon}^{L}[(1 - r_+/r)(1 - r_-/r)]^{-2}r^2\omega^3 dr \tag{20}$$

其中 l_0 满足 $\omega^2 - g l_0(l_0+1)/r^2 = 0$，把(20)式代入(17)式，得

$$F = -\frac{2}{3\pi}\int_{r_+ + \varepsilon}^{L}dr(1 - r_+/r)(1 - r_-/r)^{-2}r^2\int_0^\infty\frac{d\omega\,\omega^3}{e^{\beta\omega} - 1} = -2\pi^3/45\beta^4\{A_1L^3 +$$
$$A_2L^2 + A_3L + A_4/\varepsilon + A_5\ln[(L - r_+)/\varepsilon] + A_6\ln[(L - r_-)/(r_+ - r_-)] + A_7\} \tag{21}$$

其中，$A_1 = 1/3$，$A_2 = 2M$，$A_3 = (3r_+ + 4r_+r_- + 3r_-)$，$A_4 = r_+^6/(r_+ - r_-)^2$，

$A_5 = 2r_+^5(2r_+ - 3r_-)/(r_+ - r_-)^3$，$A_6 = 2r_-^5(3r_+ - 2r_-)/(r_+ - r_-)^3$，

$A_7 = r_-^6/(r_+ - r_-)^3 + r_+(13r_+^2)/3 + 5r_+r_- + 3r_-^2) + O(L^{-1}) + O(\varepsilon)$.

由 $S_q = \beta^2\frac{\partial F}{\partial\beta}\Big|_{\beta=\beta_H}$，$\beta_H = \frac{4\pi r_+}{1 - u}$，得

$$S_q = \frac{8\pi^3}{45\beta_H^3}\left[A_1L^3 + A_2L^2 + A_3L + \frac{A_4}{\varepsilon} + A_5\ln\frac{(L - r_+)}{\varepsilon} + A_6\ln\frac{L - r_-}{r_+ - r_-} + A_7\right] \tag{22}$$

3 讨　论

(22)式的结果和文献[8]中用泛函积分计算的结果是一致的．当 $\varepsilon\to 0$ 时，修正熵存在线性发散和指数发散．线性发散可以吸进重整化的牛顿常数中[7]，指数发散可通过在背景作用量加上项减除[8]．对于极端黑洞，$r_+ = r_-$，S_q 存在更高次的发散[8]，这不同于二维极端黑洞．

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The Quantum Correct Entropy of the Charged Black Hole

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Abstract The study of quantum correct entropy of the 2D and 4D charged black hole by " brick wall" model is reported. It is found that the results of this work are the same as those derived from the functional integral. The quantum correct entropy of the exetreme black hole is zero in 2D but divergent in 4D.

Keywords charged black hole, quantum correct entropy, " brick wall" model

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