

Larmor 钟与隧穿时间*

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摘要 计算了串联势垒系统中的 Larmor 时间, 从计算结果, 对 Larmor 时间的可加性和能否满足判据 $f_D = f(\lambda) \circ f_T + d(\lambda) \circ f_R$ 等性质作了讨论. 并阐述了关于 Larmor 时间与透射时间 f_T , 反射时间 f_R 和逗留时间 f_D 的关系的观点.

关键词 Larmor 钟, 隧穿时间, Larmor 时间

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Larmor 钟是研究隧穿时间问题的重要方法之一. 1983 年, Buttiker 对方势垒中的 Larmor 钟作了详细分析^[1], 定义了习惯上称为 Larmor 时间的 6 个量: f_T , f_{T^*} , f_{T^*} , f_R , f_{R^*} 和 f_D , 并给出 Larmor 时间与透射时间 f_T , 反射时间 f_R 和逗留时间 f_D 之间的关系. 至今已有不少人从事 Larmor 钟的研究^[2-4], 但仍未能就 Larmor 时间与 f_T , f_R 的关系给出统一意见.

1 串联势垒系统的 Larmor 钟

带 Larmor 钟的串联势垒系统 (图 1) 中势垒高度分别为 V_1, V_2 , 厚度分别为 $d_1 = b_1 - a_1, d_2 = b_2 - a_2$. 在势垒区域内有沿 z 轴的磁场 B_0 , 粒子沿 y 轴入射, 入射能量为 E , 粒子自旋取 x 轴方向. 可求得串联势垒系统中自旋向上、向下的透射、反射系数分别为 $D_{\pm} = D(\kappa_{\pm}, \kappa_{\pm})$, $A_{\pm} = A(\kappa_{\pm}, \kappa_{\pm})$, 其中, $\kappa_{\pm} = [2m(V_n - E \mp \hbar \omega_L / 2)]^{1/2} \hbar$ ($n = 1, 2$).

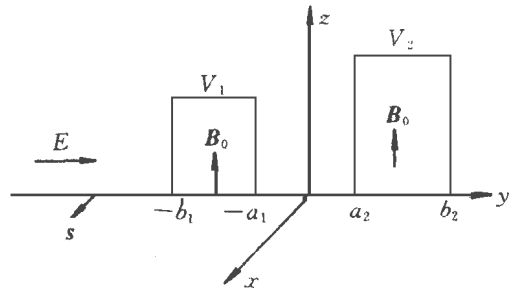


图 1 串联势垒系统的 Larmor 钟示意图
Fig. 1 Pattern of Larmor clock presented in series barriers system

$$D(\kappa_1, \kappa_2) = \frac{(T_1^* - U_1^*)(T_2^* - U_2^*)}{f_1 f_2 - g_1 g_2}, \quad A(\kappa_1, \kappa_2) = \frac{(g_1^* f_2 - f_1^* g_2)}{f_1 f_2 - g_1 g_2} \quad (1)$$

$$f_n = [U_n^2 \exp(-\kappa_n d_n) - U_n^2 \exp(\kappa_n d_n)] \exp(ik d_n),$$
$$g_n = 2T_n U_n \sinh \kappa_n d_n \exp(ik d_n), \quad T_n = U_n = (1 + ik \kappa) \quad (n = 1, 2) \quad (2)$$

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$$k = \sqrt{2mE} \hbar^{-1}, K = \sqrt{2m(V-E)} \hbar^{-1}$$

式中, 下标 “+”, “-” 分别表示自旋向上、向下的分量, $k_{\pm} = eB_0/mc$ 是 Larmor 频率, 根据 Buttiker 对 Larmor 钟的定义^[1]

$$\frac{\hbar}{2} k_L \xi_{T(R)} = \lim_{\epsilon \rightarrow 0} \langle S_x \rangle_{T(R)}, \frac{\hbar}{2} k_L \xi_{\bar{T}(R)} = \lim_{\epsilon \rightarrow 0} \langle S_y \rangle_{T(R)}, \frac{\hbar}{2} [1 - (1/2) k_L^2 \xi_{T(R)}^2]^{1/2} = \lim_{\epsilon \rightarrow 0} \langle S_x \rangle_{\bar{T}(R)} \quad (3)$$

式中, $\langle S_x \rangle_{T(R)}$, $\langle S_y \rangle_{T(R)}$, $\langle S_x \rangle_{\bar{T}(R)}$ 是电子自旋在 z, y, x 方向的分量, 可由泡利矩阵对波函数取平均给出, 可求出串联方势垒系统的 Larmor 时间

$$\xi_{T\pm} = -\frac{m}{\hbar \kappa_1} \frac{\partial \ln f(\lambda)}{\partial \kappa_1} - \frac{m}{\hbar \kappa_2} \frac{\partial \ln f(\lambda)}{\partial \kappa_2}, \xi_{\bar{T}\pm} = -\frac{m}{\hbar \kappa_1} \frac{\partial \Delta h_T}{\partial \kappa_1} - \frac{m}{\hbar \kappa_2} \frac{\partial \Delta h_T}{\partial \kappa_2}, \xi_{T\pm} = (\xi_{T+} \xi_{T-})^{1/2} \quad (4)$$

$$\xi_{R\pm} = -\frac{m}{\hbar \kappa_1} \frac{\partial \ln d(\lambda)}{\partial \kappa_1} - \frac{m}{\hbar \kappa_2} \frac{\partial \ln d(\lambda)}{\partial \kappa_2}, \xi_{\bar{R}\pm} = -\frac{m}{\hbar \kappa_1} \frac{\partial \Delta h_R}{\partial \kappa_1} - \frac{m}{\hbar \kappa_2} \frac{\partial \Delta h_R}{\partial \kappa_2}, \xi_{R\pm} = (\xi_{R+} \xi_{R-})^{1/2}$$

式中, $\tau(\lambda), \rho(\lambda)$ 分别是透射率和反射率.

$$\Delta h_T = \arctg [-Im(f_1 f_2 - g_1 g_2) / Re(f_1 f_2 - g_1 g_2)] \quad (5)$$

$$\Delta h_R = \arctg [-Im(g_1^* f_2 - f_1^* g_2) / Re(g_1^* f_2 - f_1^* g_2)] - \Delta h_T$$

2 Larmor 时间的性质

对于串联方势垒系统, 粒子会在 $[-a_1, a_2]$ 区域内振荡, 这对 Larmor 时间的影响很大, Leavens 等人曾对 $V_1 = V_2$ 时的情况作了研究^[2]. 为了方便对下以 Larmor 时间的讨论, 取 $a_1 = a_2 = 0$ 以消除粒子振荡的影响. 首先检验 Larmor 时间的可加性. 把 Larmor 钟分别应用到区域 $[-b_1, 0], [0, b_2]$ 内, 可以证明, $\xi_T, \xi_R, \xi_{\bar{T}}, \xi_{\bar{R}}$ 满足可加性, ξ_T, ξ_R 不能满足可加性. 从这一点来看, ξ_T, ξ_R 并非是有效的的时间量.

由 (4) 式可以证明:
$$f(\lambda) \xi_{T+} d(\lambda) \xi_{R-} = 0 \quad (6)$$

这与文献 [1] 相同, 反映了自旋在 z 轴方向的守恒.

由 (5), (6) 式求得

$$\xi_{\bar{T}\pm} = \frac{m f(\lambda) k_1^2}{\hbar \kappa_1 \kappa_2} \left[\frac{\kappa_2}{\kappa_1} \text{Sinh}^2 \kappa_1 b_1 \text{Sinh} 2\kappa_2 b_2 \pm \text{Sinh}^2 \kappa_2 b_2 \right] u(\kappa_2, \kappa_1, b_1) + \frac{m f(\lambda)}{\hbar \kappa_1} u(k, \kappa_1, b_1) + \frac{m f(\lambda) k_1^2}{\hbar \kappa_2 \kappa_1} \left[\frac{\kappa_1}{\kappa_2} \text{Sinh}^2 \kappa_2 b_2 \text{Sinh} 2\kappa_1 b_1 \pm \text{Sinh}^2 \kappa_1 b_1 \right] u(\kappa_1, \kappa_2, b_1) + \frac{m f(\lambda)}{\hbar \kappa_2} u(k, \kappa_2, b_2) \quad (7)$$

$$\xi_{R\pm} = \xi_{T\pm} + \frac{m f(\lambda) k_1^2}{\hbar \kappa_1 d(\lambda) \kappa_2} \left[\frac{\kappa_2}{\kappa_1} \text{Sinh}^2 \kappa_1 b_1 \text{Sinh} 2\kappa_2 b_2 \pm \text{Sinh}^2 \kappa_2 b_2 \right] u(\kappa_2, \kappa_1, b_1) - \frac{m f(\lambda) k_1^2}{\hbar \kappa_2 d(\lambda) \kappa_1} \left[\frac{\kappa_1}{\kappa_2} \text{Sinh}^2 \kappa_2 b_2 \text{Sinh} 2\kappa_1 b_1 \pm \text{Sinh}^2 \kappa_1 b_1 \right] u(\kappa_1, \kappa_2, b_2)$$

式中, $u(p, q, d) = (1 - p^2/q^2) \text{Sinh} 2qd + (1 - p^2/q^2)(2qd), k_n^2 = 2mV_n \hbar^{-2}, (n = 1, 2)$. 由 (7) 式可知: $\xi_{\bar{T}\pm} \neq \xi_{R\pm}$, 这与文献 [1] 的结果不同. 对此, 可认为: 取 $\xi_{\bar{T}\pm} = \xi_{R\pm}$ 的结论^[1] 不是普适结果, $\xi_{T\pm} = \xi_{R\pm}$ 是否成立与势垒的对称性有关.

根据 Hauge 等^[5] 的讨论, Larmor 时间与 ξ_T, ξ_R, ξ_b 三者必须满足

$$\xi_b = f(\lambda) \xi_{T+} d(\lambda) \xi_{R-} \quad (8)$$

式中, ξ_b 由 Smith^[8] 的定义给出 $\xi_b = \frac{1}{j} \int_{x_1}^{x_2} |h(x)|^2 dx$, 式中, j 是入射流, $[x_1, x_2]$ 是势垒所在区域, $h(x)$ 是系统波函数. 代入串联方势垒系统在 $B_0 = 0, a_1 = a_2 = 0$ 时的波函数可得

$$f_D = \frac{m f(\lambda)}{8 k \kappa_1} \left[\frac{4 \kappa_2 k_2^2}{\kappa_1 \kappa_2^2} \text{Sinh}^2 \kappa_1 b_1 \text{Sinh} 2 \kappa_2 b_2 + \frac{k_2^2}{\kappa_2^2} \text{Cosh} 2 \kappa_2 b_2 \right] \circ u(\kappa_2, \kappa_1, b_1) + \left[1 - \frac{k_2^2}{\kappa_2^2} \right] \circ v(\kappa_2, \kappa_1, b_1) + \frac{m f(\lambda)}{4 k \kappa_2} \circ u(k, \kappa_2, b_2) \quad (9)$$

式中, $v(p, q, d) \equiv (1 - p^2/q^2) \text{Sinh} 2qd + (1 + p^2/q^2)(2qd)$. 由 (7) 和 (9) 式可知

$$f_D \neq f_{yT}, f_D \neq f_{yR}$$

由此可认为文献 [1] 中认为自旋在 y 轴的分量决定于 f_D 的观点是不恰当的.

将 (7) 式代入 (6) 式右边的 f_T, f_R 可证明: $f_D = f(\lambda) f_{yT} + d(\lambda) f_{yR}$.

由 (4) 式可知 $f_{xT} > f_{yT}, f_{xR} > f_{yR}$, 有

$$f_D = f(\lambda) f_{yT} + d(\lambda) f_{yR} < f(\lambda) f_{xT} + d(\lambda) f_{xR} \quad (10)$$

3 讨 论

根据可加性的证明结果以及 (6), (10) 式的结果, 可认为: 由于 f_{xT}, f_{xR} 既不能满足可加性, 又不能满足必要条件 (8) 式, 故不应取 $f_T = f_{xT}, f_R = f_{xR}$. f_T, f_R 虽可满足可加性, 但它们不能满足必要条件 (8) 式, 因 f_T, f_R 也不能给出 f_T, f_R ; 根据可加性和 (8) 式两条判据, f_{yT}, f_{yR} 是最有可能给出 f_T, f_R . 不过由于在不透势垒时, 即 $\kappa_1 b_1 \gg 1, \kappa_2 b_2 \gg 1$ 时, f_{yT} 和 f_{yR} 与势垒厚度无关, 这一困难使得人们对 f_{yT}, f_{yR} 能否给出 f_T, f_R 表示怀疑. Leavens 等 [2] 对局域 Larmor 钟的研究, 为解决这一困难提供了一种解释, 但并不为人们普遍接受. 以上这些困难, 唯有待对 Larmor 钟的进一步研究及实验上的测量来解决.

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Larmor Clock and Tunneling Time

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Abstract The Larmor time in a series-barriers system is evaluated using Larmor clock method. The properties of Larmor time and the relationship between the Larmor time and the transmitted time f_T , the reflected f_R are discussed. The criteria for $f_D = f(\lambda) \circ f_T + d(\lambda) \circ f_R$ is examined with f_D representing dwell time.

Keywords Larmor clock, tunneling time, Larmor time

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