中山大学学报（自然科学版）
第 38 卷　第 3 期　ACTA SCIENTIARUM NATURALIUM UNIVERSITATIS SUNYATSENI　Vol. 38 No. 3
1999 年　5 月　May 1999

Articke ID：0529-6579（1999）03-0034-04

Soliton in the Quantum Lattice Gas Model in One Dimension*

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Abstract：The soliton solution in the quantum lattice gas model in one dimension is found. The width, the peak, the rest energy and the effective mass of the soliton are given. The condition of the soliton existing is pointed out. The dependence of the width, the peak and the energy of the soliton on the attraction energy between nearest neighbor particles are considered.

Keywords：soliton；quantum lattice gas；Heisenberg model

CLC number：O 482.51；O 175.14　　**Document code**：A

The quantum lattice gas（QLG）model was suggested by Matsubara and Matsuda[1] as that for quantum fluids such as ^{4}He．The superfluid transformation for this system occurs in the liquid state．Solitons exist in one dimensional（1-D）or quasi 1-D physics systems has attracted a lot of research interest[2]．In this paper，we consider the soliton in the QLG model by equating the model with the anisotropic Heisenberg model in a 1-D chain．The results show that solitons exist in the QLG only if nearest neighbor particles in the QLG attract each other and the attraction energy is larger than the half band-width，and the larger the attraction energy is，the more narrow the width of the soliton is，the higher the peak，the bigger the bind energy，thus the more steady the soliton．Therefore the results[3] are erroneous．

The Hamiltonian of the QLG model in a 1-D chain[4] has the form

$$H = w\sum_{i\delta} c_i^+ c_{i+\delta} + (1/2)U\sum_{i\delta} n_i^+ n_{i+\delta} \tag{1}$$

where $n_i = c_i^+ c_i$; i labels the lattice site and $i+\delta$, $\delta = \pm 1$, labels the nearest neighbors of i; c_i and c_i^+ are, respectively, destruction and creation operators at the i-th site, and satisfy the commutation relations

$$\{c_i, c_i^+\} = 1,\ c_i^2 = c_i^{+2} = 0, \text{ and } [c_i, c_j] = [c_i, c_j^+] = 0 \quad (i \neq j) \tag{2}$$

The first term of the Hamiltonian is the hopping term and the second is the interaction between particles in nearest neighbor positions．The quantity w is negative and $2|w|$ is the half band—

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width of the QLG, the parameter U may be taken to be either positive or negative, depending on whether the nearest neighbors repulse or attract each other. Using following transformation[4]

$$c_i^+ = S_i^+,\ c_i = S_i^-,\ n_i = c_i^+ c_i = 1/2 + S_i^z \tag{3}$$

where $S_i^{\pm} = S_i^x \pm iS_i^y$ and S_i^z are the spin operators at the i-th site, we can equate the Hamiltonian (1) to that of the anisotropic Heisenberg model in a 1-D chain

$$H = \frac{w}{2}\sum_{i\delta}\left(S_i^+ S_{i+\delta}^- + S_i^- S_{i+\delta}^+\right) + \frac{U}{2}\sum_{i\delta} S_i^z S_{i+\delta}^z + U\sum_i S_i^z \tag{4}$$

Meanwhile according to the Holstein-Primakoff transformation[5], i. e. $S_i^+ = \sqrt{2S - a_i^+ a_i}\,a_i$, $S_i^- = a_i^+\sqrt{2S - a_i^+ a_i}$, $S_i^z = S - a_i^+ a_i$, and taking $S = 1/2$ and correcting to the forth order in Bose operaters a_i or a_i^+ in the Hamiltonian (4), the equation of motion for a_i is

$$i\hbar\frac{\partial a_i}{\partial t} = -2Ua_i + w\sum_\delta a_{i+\delta} + U\sum_\delta a_{i+\delta}^+ a_{i+\delta} a_i -$$

$$\frac{w}{2}\sum_\delta\left(a_{i+\delta}^+ a_{i+\delta} a_{i+\delta} + a_{i+\delta}^+ a_i a_i + 2a_i^+ a_i a_{i+\delta}\right) \tag{5}$$

We use Glauber's coherent state $|\alpha>$[6], which satisfies the relation

$$a_i\,|\alpha_i> = \alpha_i\,|\alpha_i> \tag{6}$$

where α_i is the probability amplitude of the spin at the i-th site. From Eq. (3) it is clear that α_i is also the probability amplitude of the particle in the QLG model at the i－th site. Meanwhile applying the continuum limit, we obtain the equation of motion for $\alpha(x, t)$

$$i\hbar\alpha_t = 2(w - U)\alpha + wa^2\alpha_{xx} + Ua^2\alpha(|\alpha|^2)_{xx} + 2(U - 2w)\alpha|\alpha|^2 \tag{7}$$

Hereafter, the subscripts of a function denote partial derivative, and a is the lattice constant.

We now look for the solution of Eq. (7) of the form

$$\alpha(x, t) = \varphi(\eta)\exp i\left(kx - \frac{E}{\hbar}t\right) \tag{8}$$

where φ is a real function of $\eta = x - vt$. Eq. (7) becomes

$$k = -\hbar v/2wa^2, \text{ and } (1 + \gamma\varphi^2)\varphi_{\eta\eta} + \gamma\varphi\varphi_\eta^2 + 2\kappa\varphi^3 - \lambda\varphi = 0 \tag{9}$$

with

$$\gamma = \frac{2U}{w},\ \kappa = \frac{U - 2w}{wa^2},\ \lambda = \frac{2(U - w) + wk^2a^2 + E}{wa^2} \tag{10}$$

Equation (9) is a nonlinear equation and has an exact solution, i.e., the soliton solution, as

$$\frac{1}{\sqrt{\lambda}}\operatorname{arcosh}\frac{2\lambda\varphi^{-2} - (\kappa - \lambda\gamma)}{\kappa + \lambda\gamma} + \sqrt{\frac{\gamma}{\kappa}}\arcsin\frac{2\kappa\gamma\varphi^2 - (\kappa - \lambda\gamma)}{\kappa + \lambda\gamma} = 2(\eta - \eta_0) \tag{11}$$

where $\eta_0 = x_0 - vt_0$ indicates the center of the soliton being at x_0 at the time t_0. Since $a^2\kappa\gamma\varphi^2 \sim \varphi^2 \ll 1$, then an approximate soliton of Eq. (9) is

$$\varphi(\eta) = \left[\left(\frac{\kappa}{\lambda} + \gamma\right)\cosh^2\sqrt{\lambda}(\eta - \eta_0) - \gamma\right]^{-1/2} \tag{12}$$

Our following results are based on Eq. (12). As can be seen from Eq. (12), it is necessary that $\kappa > 0$. From Eq. (10), this means $U < 0$ and $|U| > 2|w|$, as $w < 0$. If either $U < 0$ but $|U| < 2|w|$ or $U > 0$, the soliton can not exist. As a result, the soliton can only exist in the QLG in which nearest neighbors attract each other and the attraction energy is larger than the half

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hand-width. If not, the soliton can not exist. These are different from the results[3] in which the method used is erroneous and the soliton existing only if the attraction energy is very small is concluded. In fact, the QLG model may be equated to an anisotropic Heisenberg model represented by Eq. (4). When $U > 0$, Eq. (4) describes an anisotropic antiferromagnetic chain in which the soliton can not exist. When $U < 0$ but $|U| < 2|w|$, although Eq. (4) describes an anisotropic ferromagnetic chain, but as mentioned[5], its groundstate is instable.

The normalization for α leads to Eq. (12) rewritten as

$$\varphi(\eta) = \left[\gamma \operatorname{cosec}^2 \frac{\sqrt{\kappa\gamma}a}{2} \cosh^2 \frac{\eta - \eta_0}{L_s} - \gamma\right]^{-1/2} \tag{13}$$

where $L_s = a\sqrt{\dfrac{2U}{U-2w}}\cot\dfrac{\sqrt{2U(U-2w)}}{-2w}$, is the width of solitons. The peak of solitons is

$$P_s = \sqrt{\frac{w}{2U}}\tan\frac{\sqrt{2U(U-2w)}}{-2w} \tag{14}$$

From Eqs. (9), (10) and (13), we get the energy of solitons

$$E_s = 2(w-U) - E_b + (1/2)m_s v^2 \tag{15}$$

the first term $2(w-U)$ is the energy gap, and $E_b = -\dfrac{w(U-2w)}{2U}\tan^2\dfrac{\sqrt{2U(U-2w)}}{-2w}$ is the energy lowered by the nonlinear interactions resulting in solitons in the Hamiltonian (5), and may be viewed as the bind energy of solitons; while

$$m_s = -\hbar^2/2wa^2 \tag{16}$$

is the effective mass of solitons. From Eqs. (8) and (13), the probability of a particle at x is

$$\alpha(x, t) == \left[\gamma \operatorname{cosec}^2 \frac{\sqrt{\kappa\gamma}a}{2} \cosh^2 \frac{(x-x_0)-(t-t_0)}{L_s} - \gamma\right]^{-1/2} \exp i\left(kx - \frac{E_s}{\hbar}t\right) \tag{17}$$

From Eqs. (14) ~ (16), we can easily show if $|U| > 2|w|$, then $\mathrm{d}L_s/\mathrm{d}|U| < 0$, $\mathrm{d}P_s/\mathrm{d}|U| > 0$ and $\mathrm{d}E_b/\mathrm{d}|U| > 0$. That is to say, the larger the attraction energy is, the more narrow the width of the soliton is, the higher the peak, the bigger the bind energy, and consequently the more steady the soliton.

In summary, we found the soliton solution of the QLG model in 1-D and considered the condition of the soliton existing and the dependences of the width, the peak and the energy of the soliton on the attraction energy between nearest neighbor particles in the QLG. The results obtained above show that solitons can exist in the QLG only if nearest neighbor particles attract each other and the attraction energy is larger than the half band-width. And the fact, solitons will be more steady when the attraction energy is larger, is also proven. Meanwhile when the attraction energy between nearest neighbors is equivalent to the half band-width of the QLG, i.e. $|U| = 2|w|$, we can see, from Eqs. (14) and (17), $\alpha = 0$, that is, there is no the soliton. It is because the nonlinear interaction terms in the Eq. (7) are vanished and the attraction and hopping are in equilibrium for this case. In fact, in this case, from Eqs. (10) and (13), we obtain $\kappa = \lambda = 0$. So the dispersion relation of excitations is $E = -2w - 2wa^2k^2$, which is that of spin waves with a gap $-2w$[7]. Therefore solitons are reduced to spin waves.

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一维量子格气模型中的孤子

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摘　要：得到了一维量子格气模型中的孤子解，给出了孤子的宽度、峰值、静止能量及有效质量．结果表明孤子的存在是有条件的，孤子的宽度、峰值及其能量依赖于最近邻粒子间的相互吸引能.

关键词：孤子；量子格气；海森堡模型
中图分类号：O 482.51；O 175.14　　文献标识码：A

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