中山大学学报 (自然科学版)
ACTA SCIENTIARUM NATURALIUM UNIVERSITATIS SUNYATSENI
Vol.39 Suppl. (2)
Dec. 2000

文章编号: 0529-6579 (2000) S2-0001-05

二阶多色波对结构的绕射问题*

黄　华，詹杰民，朱顶金，廖德驹
(中山大学应用力学与工程系，广州 510275)

摘　要: 引入二层海的层化海模型及二阶水波的多色波作用机制，对层化海条件下的二阶多色波的绕射作用问题进行了分析．推导了二层海二阶绕射波的远场解式．指出了二层海条件下二阶绕射波作用积分解建立的数学过程．

关键词: 二层海；层化海；二阶波；多色波；绕射；散射；波浪力
中图分类号: O353　　**文献标识码**: A

层化海洋的水波特性已明显不同于均匀海下的单色波情形．多色波的作用机制对结构的波浪力及水动力学系数有重要影响[1,2]．如果再计入柱群结构的波干扰以及波的非线性等因素，则层化海的水波绕射问题将更趋复杂．在层化海条件下，流场中存在层流面，二阶波具有和、差频不同分量影响，单色波理论已不再适用．为了简化分析，并考虑到层化海内波能量主要集中于最低型内波上[1,3]，以二层海为模式进行初步研究，其结果将具有广泛意义．对此，本文按波分解法，求解了二层海中多色波二阶散射分量在流场远区的渐近解，并由积分法取得了二阶波散射作用积分解的数学形式．

1　多色波绕射问题的边值方程

引入水波势特征解法[4]，对流域（图 1）按 v_1-v_3 进行分区，再对各柱分别建立柱坐标 $(r_e, \theta_e, z)(e = 0,1,\cdots,k_c; e = 0$ 对应底柱, k_c 为顶柱总数)．设二层海水密度分别为上层 ρ_2 及下层 ρ_1 且 $\rho_2 > \rho_1$, 令 $z = h_0$ 为其层流界面，其位置有 3 种可能情形：① $h_1 + h_2 < h_0 \leqslant h$（图 1)；② $h_1 < h_0 \leqslant h_1 + h_2$；③ $0 \leqslant h_0 \leqslant h_1$．再令 $\Phi_1, \Phi_2, \varphi_m^i, \varphi_m^s, \varphi_{inm}^{\pm}$ 及 $\varphi_{snm}^{\pm}$ 分别为水波一阶势、二阶势及相应一、二阶入射、散射波势分量，相应边值方程为：

$$\Phi_1 = \Phi_1^i + \Phi_1^s = \sum_{m=1} \{ Re[\varphi_m^i e^{-i\omega_m t}] + Re[\varphi_m^s e^{-i\omega_m t}] \} \tag{1a}$$

$$\Phi_2 = \Phi_2^i + \Phi_2^s = \sum_{n,m=1} [(\varphi_{inm}^{+} + \varphi_{snm}^{+}) e^{-i\Omega_{nm}^{+} t} + (\varphi_{inm}^{-} + \varphi_{snm}^{-}) e^{-i\Omega_{nm}^{-} t}] \tag{1b}$$

$$\nabla^2(\varphi_m^s; \varphi_{snm}^{\pm}) = 0 \quad (v_1 + v_2 + v_3); \frac{\partial}{\partial z}(\varphi_m^s; \varphi_{snm}^{\pm}) = 0 \quad (z = 0) \tag{2a}$$

---

* 基金项目: 国家自然科学基金资助项目 (19802023)
收稿日期: 2000-08-14;　　作者简介: 黄华 (1961～), 男, 副教授.

$$\frac{\partial}{\partial n}[(\varphi_m^s+\varphi_m^i);(\varphi_{snm}^{\pm}+\varphi_{inm}^{\pm})]=0(s_b);\lim_{r_0\to\infty}\sqrt{r_0}\left(\frac{\partial}{\partial r_0}\varphi_m^s-ik\varphi_m^s\right)=0 \tag{2b}$$

$$\frac{\partial}{\partial z}(\varphi_m^s;\varphi_{snm}^{\pm})-[\omega_m^2\varphi_m^s;(\Omega_{nm}^{\pm})^2\varphi_{snm}^{\pm}]=(0;A_{nm}^{\pm})(z=h) \tag{2c}$$

$$g\rho\left\{g\frac{\partial}{\partial z}(\varphi_m^s;\varphi_{snm}^{\pm})-[\omega_m^2\varphi_m^s;(\Omega_{nm}^{\pm})^2\varphi_{snm}^{\pm}]\right\}\Bigg|_{z=h_0-0}^{z=h_0+0}=(0;B_{nm}^{\pm}) \tag{2d}$$

$$\frac{\partial}{\partial z}(\varphi_m^s;\varphi_{snm}^{\pm})\bigg|_{z=h_0+0}-\frac{\partial}{\partial z}(\varphi_m^s;\varphi_{snm}^{\pm})\bigg|_{z=h_0-0}=(0;C_{nm}^{\pm}) \tag{2e}$$

式中，$\Omega_{nm}^{\pm}=\omega_n\pm\omega_m$ 为二阶和、差频；二阶辐射条件模式待定；s_b 为结构表面；$A_{nm}^{\pm}$，$B_{nm}^{\pm}$ 及 $C_{nm}^{\pm}$ 为一阶入射与散射势耦合项所构成的强迫项；令 $T=\tanh kh$，$T_1=\tanh k(h-h_0)$，$T_2=\tanh kh_0$；$\rho_0=\rho_2/\rho_1$，则参数 $k=k_m$ 为以下色散方程当 $\omega=\omega_m$ 的正实根：

$$\omega^2=\frac{k\{\rho_0T(1+T_1T_2)\pm[\rho_0^2T^2(1+T_1T_2)^2-4(\rho_0-1)T_1T_2(\rho_0+T_1T_2)]^{1/2}\}}{2(\rho_0+T_1T_2)} \tag{3}$$

式中的第1、2解分别对应表面波和内波.

2 多色波一阶解与二阶远场解

一阶入射波势在二层海模式下的数学解可写为[1]：

$$\varphi_m^i=i(k_m\omega_m)^{-1}A_mf_1(\omega_m,k_m,z)\cosh k_mz\cosh k_mh\mathrm{e}^{ik_mx} \tag{4a}$$

$$f_1(\omega,k,z)=\begin{cases}k-vT+(v-kT)\tanh kz & (h_0<z\leqslant h)\\ (v-kT_1)(1-TT_2)/T_2 & (0\leqslant z<h_0)\end{cases} \tag{4b}$$

其中，ω，k 满足色散方程(3)；$v=\omega^2/g$；A_m 为对应 ω_m 的波振幅.现取：

$$\begin{aligned}\varphi_m^{(1)}=\varphi_m^{i(1)}+\varphi_m^{s(1)}=\sum_{n=-\infty}^{\infty}[&i^{n+1}(k_m\omega_m)^{-1}A_m\cosh k_mhf_1(\omega_m,k_m,z)\cosh k_mz\times\\ &J_n(k_mr_0)\mathrm{e}^{in\theta_0}+A_{nlm}^{(1)}K_n(\alpha_{lm}^{(1)}r_0)\cos\alpha_{lm}^{(1)}zf_1(\omega_m,i\alpha_{lm}^{(1)},z)\mathrm{e}^{in\theta_0}]\end{aligned} \tag{5}$$

$$\varphi_m^{(2)}=\varphi_m^{i(2)}+\varphi_m^{s(2)}=\sum_{n=-\infty}^{\infty}\sum_{l=0}^{\infty}P_{nl}^{(2)}(r_0,z)\mathrm{e}^{in\theta_0} \tag{6a}$$

$$\begin{aligned}p_{nl}^{(2)}(r_0,z)=&(\delta_{1c}+\delta_{2c})A_{nlm}^{(2)}[\delta_{0l}r_0^n+2(1-\delta_{0l})I_n(l\pi h_1^{-1}r_0)]\cos l\pi h_1^{-1}z+\\ &\delta_{3c}A_{nlm}^{(2)}f_2(z)I_n(\alpha_{lm}^{(2)}r_0)\end{aligned} \tag{6b}$$

$$f_2(z)=\begin{cases}[(v_m-kT_1')(1-T'T_2')/T_2']\cos\alpha_{lm}^{(2)}(h_1-z) & (\alpha_{lm}^{(2)}=\beta_{lm};h_0<z\leqslant h)\\ f_1(\omega_m,i\alpha_{lm}^{(2)},z) & (\alpha_{lm}^{(2)}\text{ 为 }\alpha_{lm}^{(1)}\text{ 的内波解};0\leqslant z<h_0)\end{cases} \tag{6c}$$

$$\varphi_m^{(3)}=\varphi_m^{i(3)}+\varphi_m^{s(3)}=\sum_{n=-\infty}^{\infty}\sum_{l=0}^{\infty}\sum_{f=1}^{k_c}p_{nl}^{(3)}(r_f,z)\mathrm{e}^{in\theta_f} \tag{7a}$$

$$\begin{aligned}p_{nl}^{(3)}(r_f,z)=&(\delta_{3c}+\delta_{2c})[A_{nlm}^{3,f}I_n(\mu_{lm}r_f)+\delta_{ef}B_{nlm}^{(3)}K_n(\mu_{lm}r_f)]\cos\mu_{lm}(z-h_1-h_2)+\\ &\delta_{1c}f_3(z)\cos\alpha_{lm}^{(3)}(z-h_1-h_2)[A_{nlm}^{(3,f)}I_n(\alpha_{lm}^{(3)}r_f)+\delta_{ef}B_{nlm}^{(3)}K_n(\alpha_{lm}^{(3)}r_f)]\end{aligned} \tag{7b}$$

$$f_3(z)=\begin{cases}k-v_mT''+i(v_m-kT'')\tan\alpha_{lm}^{(3)}(z-h_1-h_2) & (h_0<z\leqslant h)\\ (v_m-kT_1'')(1-T''T_2'')/T_2'' & (h_1+h_2\leqslant z<h_0)\end{cases} \tag{7c}$$

式中，$\varphi_m^{(k)}$ 表示 v_k 区内的一阶总势；$c=1,2,3$ 分别代表层流面 1) ~ 3) 的 3 种位置状态；

$A_{nlm}^{(1)}, A_{nlm}^{(2)}$、$A_{nlm}^{(3,f)}$ 及 $B_{nlm}^{(3)}$ 为待定系数；$J_n(x), I_n(x)$ 及 $K_n(x)$ 为各类 Bessel 函数；另有：$\alpha_{0m}^{(1)} = -ik_m, \alpha_{lm}^{(1)}(l \neq 0)$ 为对应色散方程(3)的虚根；$T' = -\tanh \beta_{lm}(h-h_1), T_2' = \tanh \beta_{lm} h_0$, $T_1' = \tanh \beta_{lm}(h_1 - h_0), T_1'' = \mathrm{i} \tan \alpha_{lm}^{(3)}(h_0 - h_1 - h_2)$，其中以 T', T_1' 和 T_2' 代替 T, T_1 和 T_2，则 β_{lm} 为满足色散方程(3)的内波解；$T'' = \mathrm{i} \tan \alpha_{lm}^{(3)}(h - h_1 - h_2), T_2'' = \mathrm{i} \tan \alpha_{lm}^{(3)} h_0$，其中以 T'', T_1'' 和 T_2'' 代替 T, T_1 和 T_2，则 $\alpha_{lm}^{(3)}$ 为满足色散方程(3)的实、虚根；$\mu_{lm} = \tan \mu_{lm}(h - h_1 - h_2) = -\omega_m^2/g$. 可证：式中 $\varphi_m^{(1)}, \varphi_m^{(2)}$ 和 $\varphi_m^{(3)}$ 关于 z 变量函数分别在区段 $0 \leqslant z \leqslant h, 0 \leqslant z \leqslant h_1$ 和 $h_1 + h_2 \leqslant z \leqslant h$ 上正交. 推广 Bessel 和 Graf 加法定理，有：

$$B_n^{(q)}(br_f)\{e^{in\theta_f}; \cos n\theta_f\} = \sum_{j=-\infty}^{\infty}[\delta_{1q}(-1)^j + \delta_{2q}] B_{n+j}^{(q)}(b\, {}^{r_{ef}}_{e}) B_j^{(q-\delta_{2q})}(b\, {}^{r_{ef}}_{e}) \times$$
$$\{\exp(in\theta_{fe} + ij(\theta_{ef} - \theta_e)); \cos[j\theta_e - n\pi - (j+n)\theta_{ef}]\}$$
$$(\text{当 } e \neq 0 \text{ 时}, r_{ef} \geqslant r_e \geqslant a_e, r_e \geqslant r_{ef} \geqslant a_e; \text{当 } e = 0 \text{ 时}, r_{ef} > r_e, r_e \geqslant r_{ef}) \tag{8}$$

式中，$B_n^{(q)}(x)$ 为广义 Bessel 函数，$q = 1,2$ 分别对应 $I_n(x)$ 和 $K_n(x)$，b 为一般性参数；通过此定理，即可实现流场内域 v_3 中各柱坐标间变换，再由各径向物面条件与各区交界面连续条件，并通过特征解正交性作傅利叶级数展开[4]，便可建立待定系数的完备代数方程组.

对二阶波散射势及强迫项作分解并变换层流面边值条件，可得：

$$\varphi_{snm}^{\pm} = \varphi_{snm1}^{\pm} + \varphi_{snm2}^{\pm}; \quad \nabla^2(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm}) = 0 \quad (v_1 + v_2 + v_3) \tag{9}$$

$$A_{nm}^{\pm} = A_{nm}^{\pm(i,s)} + A_{nm}^{\pm(s,s)}, B_{nm}^{\pm} = B_{nm}^{\pm(i,s)} + B_{nm}^{\pm(s,s)}, C_{nm}^{\pm} = C_{nm}^{\pm(i,s)} + C_{nm}^{\pm(s,s)} \tag{10}$$

$$g\frac{\partial}{\partial z}(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm}) - (\Omega_{nm}^{\pm})^2(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm}) = (A_{nm}^{\pm(i,s)}; A_{nm}^{\pm(s,s)}) \quad (z = h) \tag{11a}$$

$$(\rho_1 - \rho_2)\left[g\frac{\partial}{\partial z}(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm} - (\Omega_{nm}^{\pm})^2(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm})\right]\Bigg|_{z = h_0 - 0} =$$
$$(B_{nm}^{\pm(i,s)} - \rho_1 g C_{nm}^{\pm(i,s)}; B_{nm}^{\pm(s,s)} - \rho_1 g C_{nm}^{\pm(s,s)}) \tag{11b}$$

式中，$A_{nm}^{\pm(i,s)} \sim C_{nm}^{\pm(i,s)}$ 及 $A_{nm}^{\pm(s,s)} \sim C_{nm}^{\pm(s,s)}$ 分别对应 φ_n^s 与 $\varphi_m^{i,\pm}$ 及 φ_n^s 与 $\varphi_m^{s,\pm}$ 的耦合作用项；再令：

$$\nabla^2(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm}) = o(r_0^{-1/2})(k_n r_0 \gg 1); \frac{\partial}{\partial z}(\varphi_{snm1}^{\pm}; \varphi_{snm2}^{\pm}) = 0 \quad (z = 0) \tag{12}$$

对多色波一阶解取渐近式，由渐近法，可得：

$$\varphi_{snm1}^{\pm} + \varphi_{snm2}^{\pm} \sim r_0^{-1/2}\left\{\begin{pmatrix} D_1^{\pm}(\theta) \\ E_1^{\pm}(\theta) \end{pmatrix} \cosh\left[k_n(1 \pm 2\, b_{nm}\cos\theta + b_{nm}^2)^{1/2} z\right] \times\right.$$
$$\left.\exp(\mathrm{i} r_0 k_n(1 \pm b_{nm}\cos\theta)) + \begin{pmatrix} D_2^{\pm}(\theta) \\ E_2^{\pm}(\theta) \end{pmatrix} \cosh k_{nm}^{\pm} z \exp(\mathrm{i} k_{nm}^{\pm} r_0)\right\}$$
$$\begin{pmatrix} h \geqslant z > h_0 \\ h_0 \geqslant z > 0 \end{pmatrix}, k_n r_0 \gg 1 \tag{13}$$

式中，$b_{nm} = k_m/k_n$；$k_{nm}^{\pm}$ 为 $\omega = \Omega_{nm}^{\pm}$ 时色散方程(3)的正实根；$D_t^{\pm}(\theta)$ 与 $E_t^{\pm}(\theta)(t = 1,2)$ 分别由自由面及层流面条件的渐近式确定．易证，所给二阶远场解已满足散射势各分量边值问题的渐近方程式，其振动衰减特性与单色波相同，是二阶辐射条件的一种可行模式．

3　二阶多色波散射作用的积分解

单色波的二阶散射势直接求解迄今仍为一个较复杂的问题，对多色波而言更是如此．

然而由积分法仍可取得对多色波二阶散射作用的一种算式．为方便计，仅考虑结构水平方向波力计算，其它波力（矩）计算方式相同．令 $F_{nm}^{\pm}$为第二阶散射水平波力，有：

$$F_{nm}^{\pm} = D_{nm}^{\pm}\iint_{S_b}\rho\varphi_{snm}^{\pm}n_a\mathrm{d}s = D_{nm}^{\pm}I_{nm}^{\pm} \tag{14}$$

式中，$D_{nm}^{\pm}$ 为复常数；引入线性辅助势 $\Psi_{nm}^{\pm}$ 的一组边值方程：

$$\nabla^2\Psi_{nm}^{\pm} = 0 \quad (v_1 + v_2 + v_3); \quad \frac{\partial}{\partial z}\Psi_{nm}^{\pm} = 0 \quad (z = 0) \tag{15a}$$

$$g\frac{\partial}{\partial z}\Psi_{nm}^{\pm} - (\Omega_{nm}^{\pm})^2\Psi_{nm}^{\pm} = 0 \quad (z = h)\ ;\rho\left[\frac{\partial}{\partial z}\Psi_{nm}^{\pm} - (\Omega_{nm}^{\pm})^2\Psi_{nm}^{\pm}\right]\Bigg|_{z=h_0-0}^{z=h_0+0} = 0;$$

$$\frac{\partial}{\partial z}\Psi_{nm}^{\pm}\Big|_{z=h_0+0} - \frac{\partial}{\partial z}\Psi_{nm}^{\pm}\Big|_{z=h_0-0} = 0;\lim_{r_0\to\infty}\sqrt{r_0}\left(\frac{\partial}{\partial r_0}\Psi_{nm}^{\pm} - ik_{nm}^{\pm}\Psi_{nm}^{\pm}\right) = 0 \tag{15b}$$

$$\frac{\partial}{\partial n}\Psi_{nm}^{\pm} = n_a = \frac{\partial}{\partial r_e}\Psi_{nm}^{\pm} = \delta_{1t}\cos\theta_e \quad (s_{re};t = 0,1);\frac{\partial}{\partial z}\Psi_{nm}^{\pm} = 0 \quad (s_{ve}) \tag{15c}$$

其中，s_{re}，s_{ve} 分别表示 e 号柱的径向与垂直表面；另规定，当所算波载中计入对 e 号柱水平力时，取 $t = 1$，反之，取 $t = 0$，由此可计算各单柱或各柱群组所受水平总波浪力．引入格林第二积分关系及关于 $\varphi_{snm}^{\pm}$ 和 $\Psi_{nm}^{\pm}$ 的边值方程各式，可得：

$$I_{nm}^{\pm} = \iiint_{v_1+v_2+v_3}\rho(\varphi_{snm}^{\pm}\nabla^2\Psi_{nm}^{\pm} - \Psi_{nm}^{\pm}\nabla^2\varphi_{snm}^{\pm})\mathrm{d}v - \iint_{s_b}\rho\frac{\partial\varphi_{inm}^{\pm}}{\partial n}\Psi_{nm}^{\pm}\mathrm{d}s +$$

$$\iint_{s_0+s_\infty+s_f+s_i}\rho\left(\Psi_{nm}^{\pm}\frac{\partial\varphi_{snm}^{\pm}}{\partial n} - \frac{\partial\Psi_{nm}^{\pm}}{\partial n}\varphi_{snm}^{\pm}\right)\mathrm{d}s =$$

$$-\iint_{s_b}\rho\frac{\partial\varphi_{inm}^{\pm}}{\partial n}\Psi_{nm}^{\pm}\mathrm{d}s + \rho_1 g^{-1}\iint_{s_f}\Psi_{nm}^{\pm}A_{nm}^{\pm}\mathrm{d}s - [g/(\Omega_{nm}^{\pm})^2(\rho_2 - \rho_1)]\times$$

$$\iint_{s_i}\left[(\rho_2 - \rho_1)\frac{\partial\Psi_{nm}^{\pm}}{\partial z}B_{nm}^{\pm} + \rho_1\rho_2 g^{-1}(\Omega_{nm}^{\pm})^2\Psi_{nm}^{\pm}\Bigg|_{z=-h_0-0}^{z=-h_0+0}\right]\mathrm{d}s \tag{16}$$

式中，s_0，s_∞，s_f 及 s_i 分别对应海底、无穷远垂直柱面、自由表面和二层海层流面．而 s_∞ 面积分项可由二阶远场解及驻相法证明其值趋近零．现对 $\Psi_{nm}^{\pm}$ 求解，取：

$$\Psi_{nm}^{\pm(1)} = \sum_{l=0}^{\infty}\alpha_{nm,l}^{(1)}K_1(\alpha_{lnm}^{(1)\pm}r_0)\cos\alpha_{lnm}^{(1)\pm}zf_1(\Omega_{nm}^{\pm}, i\alpha_{lnm}^{(1)\pm}, z)\cos\theta_0 \tag{17}$$

$$\begin{aligned}\Psi_{nm}^{\pm(2)} = \sum_{l=0}^{\infty}\alpha_{nm,l}^{(2)}\cos\theta_0\{&\delta_{3c}g_2(z)I_1(\alpha_{lnm}^{(3)\pm}r_0) + \\ &(\delta_{1c} + \delta_{2c})[\delta_{0l}r + 2(1 - \delta_{0l})I_1(l\pi h_1^{-1}r_0)]\cos l\pi h_1^{-1}z\}\end{aligned} \tag{18}$$

$$\Psi_{nm}^{\pm(3)} = \sum_{n_1=0}^{\infty}\sum_{l=0}^{\infty}\sum_{f=1}^{k_c}[(\delta_{2c} + \delta_{3c})h_{n_1l}(\mu_{lnm}^{\pm}, r_f, \theta_f, z) + \delta_{1c}g_3(z)h_{n_1l}(\alpha_{lnm}^{(3)\pm}, r_f, \theta_f, z)] \tag{19a}$$

$$\begin{aligned}h_{n_1l}(b, r_f, \theta_f, z) = [&(a_{nm,n_1l}^{3,f}\cos n_1\theta_f + ib_{nm,n_1l}^{3,f}\sin n_1\theta_f)I_{n_1}(br_f) + \\ &\delta_{ef}(c_{nm,n_1l}^{(3)}\cos n_1\theta_f + id_{nm,n_1l}^{3}\sin n_1\theta_f)K_{n_1}(br_f)]\cos b(z - h_1 - h_2)\end{aligned} \tag{19b}$$

式中，$\Psi_{nm}^{\pm(k)}$ 表示对应 v_k 区域的积分辅助势；$a_{nm,l}^{(k)}$，$b_{nm,l}^{(k)}$，$a_{nm,n_1l}^{(3,f)}$，$b_{nm,n_1l}^{(3,f)}$，$c_{nm,n_1l}^{(3)}$ 及 $d_{nm,n_1l}^{(3)}$ 为对应各级数解的待定系数；$\alpha_{0nm}^{(1)\pm} = -ik_{nm}^{\pm}$，而 $\alpha_{lnm}^{(1)\pm}(l \neq 0)$ 为 $\omega = \Omega_{nm}^{\pm}$ 时对应色散方程(3)的虚根；$g_2(z)$ 和 $g_3(z)$ 形式分别与 $f_2(z)$ 和 $f_3(z)$ 一致，只须以参数 $\Omega_{nm}^{\pm}$，$\alpha_{lnm}^{(k)\pm}$，$\beta_{lnm}^{\pm}$ 分别替

代原参数 $\omega_m^{\pm}, \alpha_{lm}^{(k)}, \beta_{lm}$ 即可；b 为一般性参数；另有：$\mu_{lm}^{\pm}\tan \mu_{lm}^{\pm}(h - h_1 - h_2) = -(\Omega_{nm}^{\pm})^2/g$. 而坐标间的变换可应用 Graf 公式推广式(8)进行. 对于 $\sin n_1\theta_f$ 相关项，只须将式(9)中的余弦项替换为正弦项即可. 待定系数代数方程组的建立类似一阶势求法.

本文采用层化海的基本模式——二层海模式，由多色波理论，分析了二阶多色波对结构的绕射作用问题. 提出并论证了二层海中二阶波远场解（即二阶辐射条件）的数学模式. 利用此条件及格林积分原理，得到了二阶多色波对垂直圆柱散射作用的的积分解式. 由于对一阶势特征解法加以了推广，使积分解可适用于非轴对称型圆柱的解析计算.

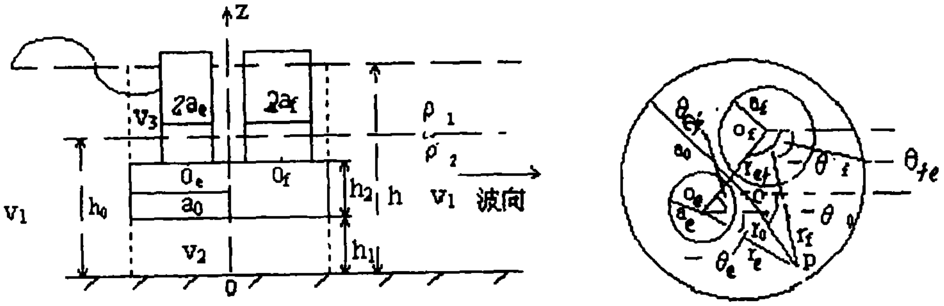

图 1 二层海中的垂直圆柱结构

参考文献：

[1] YIH C S. Stratified flows. New York: Academic Press Inc, 1980. 237～241.

[2] 吴建华，吴秀恒，李世漠. 大尺度物体在层化海洋中的波浪绕射和散射理论（Ⅰ）浮体在层化海洋中的运动方程. 水动力学研究与进展，1989, 4 (2): 81～88.

[3] WU J H. The second order wave loads in stratified ocean. Acta Mechanica Sinica, 1991, 7 (1): 24～27.

[4] 黄华，张涤明，孙明光. 有限振幅波对复合圆柱绕射波浪力的实用算法. 中山大学学报（自然科学版），1996, 35 (5): 33～38.

Problems of the Second Order Multicolored Water Wave Diffraction on Offshore Structures

HUANG Hua *, *ZHAN Jie-min*, *ZHU Ding-jin*, *LIAO De-ju*

Abstract: By introducing one of stratified ocean models: two layer ocean and multicolored wave mechanism of second order water wave, the problems of diffracting effects caused by second order multicolored wave are analyzed. The far-field solutions to second order wave in two layer ocean are derived and the mathematical analysis process of getting the integral solutions of second order wave forces is presented.

Keywords: two layer ocean; stratified ocean; second order wave; multicolored wave; diffracting; scattering; wave force

* Department of Applied Mechanics and Engineering, Zhongshan University, Guangzhou 510275, China