

九次等变平面 Hamilton 向量场的一般形式*

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摘 要: 利用旋转对称群理论, 给出了九次等变 Hamilton 向量场在复坐标和直角坐标下的两种一般形式, 为进一步研究该类向量场的各种动力学性质奠定了基础.

关键词: 等变向量场; 旋转对称群; 一般形式

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等变平面 Hamilton 向量场在弹性力学、天体物理等许多领域内都有应用. 由于其形式较简单直观并具有旋转对称性^[1], 所以它是讨论分支理论的基础, 并且往往成为研究多项式系统的极限环个数及其分支的出发点, 成为研究 Hilbert 16 问题及弱 Hilbert 16 问题的基础. 因此研究高次等变平面 Hamilton 向量场有其理论和应用价值.

李继彬、刘正荣给出了具有等变性质的平面五次 Hamilton 向量场的一般性质^[2], 讨论了具有等变性质的平面五次 Hamilton 向量场的分支^[3].

考虑实平面微分系统:

$$\frac{dx}{dt} = P(x, y), \quad \frac{dy}{dt} = Q(x, y) \quad (1)$$

P, Q 关于 x 和 y 足够光滑 (或者是 x, y 的多项式).

令 $z = x + iy, \bar{z} = x - iy$ 则 (1) 变成:

$$\frac{dz}{dt} = F(z, \bar{z}), \quad \frac{d\bar{z}}{dt} = \overline{F(z, \bar{z})} \quad (2)$$

其中, $F(z, \bar{z}) = P(u, v) + iQ(u, v), u = (z + \bar{z})/2, v = (z - \bar{z})/2i$.

显然, 当 $q \geq m+1$ 时次数为 $m-1$ 的 z_q^- 等变哈密顿系统是一个平凡的旋转不变系统.

定理 1 对平面九次 Hamilton 系统, 非平凡 z_q^- 等变向量场有如下形式:

① $q = 10$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + A_6 \bar{z}^9$$

② $q = 9$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + A_6 \bar{z}^8$$

③ $q = 8$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + (A_6 - 9\bar{A}_7 |z|^2)\bar{z}^7 + A_7 \bar{z}^9$$

④ $q = 7$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + A_6 \bar{z}^8 + (A_7 - 8\bar{A}_6 |z|^2)\bar{z}^6$$

⑤ $q = 6$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + (A_6 + A_7 |z|^2)\bar{z}^7 + (A_8 - 7\bar{A}_6 |z|^2 - 4\bar{A}_7 |z|^4)\bar{z}^5$$

⑥ $q = 5$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + (A_6 + A_7 |z|^2)\bar{z}^6 + (A_8 - 6\bar{A}_6 |z|^2 - 7/2\bar{A}_7 |z|^4)\bar{z}^4 + A_9 \bar{z}^9$$

⑦ $q = 4$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + (A_6 + A_7 |z|^2 + A_8 |z|^4)\bar{z}^5 + A_9 \bar{z}^9 + (A_{10} - 5\bar{A}_6 |z|^2 - 3\bar{A}_7 |z|^4 - 7/3\bar{A}_8 |z|^6)\bar{z}^3 + (A_{11} - 9\bar{A}_9 |z|^2)\bar{z}^7$$

⑧ $q = 3$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + (A_6 + A_7 |z|^2 + A_8 |z|^4)\bar{z}^4 + (A_9 + A_{10} |z|^2)\bar{z}^7 + (A_{11} - 4\bar{A}_6 |z|^2 - 5/2\bar{A}_7 |z|^4 - 2\bar{A}_8 |z|^6)\bar{z}^2 + (A_{12} - 7\bar{A}_9 |z|^2 - 4\bar{A}_{10} |z|^4)\bar{z}^5 + A_{13} \bar{z}^8$$

⑨ $q = 2$,

$$F(z, \bar{z}) = (b_1 + b_2 |z|^2 + b_3 |z|^4 + b_4 |z|^6 + b_5 |z|^8)zi + (A_6 + A_7 |z|^2 + A_8$$

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$$|z|^4 + A_9 |z|^6) z^3 + (A_{10} + A_{11} |z|^2 + A_{12} |z|^4) z^5 + (A_{13} + A_{14} |z|^2) z^7 + A_{15} z^9 + (A_{16} - 3\bar{A}_6 |z|^2 - 2\bar{A}_7 |z|^4 - 5/3\bar{A}_8 |z|^6 - 3/2\bar{A}_9 |z|^8) \bar{z} + (A_{17} - 5\bar{A}_{10} |z|^2 - 3\bar{A}_{11} |z|^4 - 7/3\bar{A}_{12} |z|^6) \bar{z}^3 + (A_{18} - 7\bar{A}_{13} |z|^2 - 4\bar{A}_{14} |z|^4) \bar{z}^5 + (A_{19} - 9\bar{A}_{15} |z|^2) \bar{z}^7 + A_{20} \bar{z}^9 \sum_{l=0} h_l (|z|^2) z^{l+1} \quad (3)$$

其中, $g_l(u)$, $h_l(u)$ 是 u 的复系数多项式. 进一步, 由 (2) 式定义的向量场是 z_q -等变哈密尔顿的当且仅当下式成立:

$$\frac{\partial F}{\partial z} + \frac{\partial \bar{F}}{\partial \bar{z}} \equiv 0 \quad (4)$$

由 (2) 式定义的向量场是 z_q -等变的当且仅当函数 $F(z, \bar{z})$ 有方程 (3) 的形式, 并且函数 g_l 和 h_l 的所有系数是实数.

由复数的性质, 定理得证.

定理 2 在直角坐标下, 具有等变性质的九次 Hamilton 向量场的一般形式如下:

其中, b_j 为实数, A_j 为复数.

证明 由文[3]可知: 由 (2) 式定义的向量场是 z_q -等变的当且仅当函数 $F(z, \bar{z})$ 有如下形式

$$F(z, \bar{z}) = \sum_{l=1} g_l (|z|^2) \bar{z}^{l-1} +$$

(1) $q = 10$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(9x^8y - 84x^6y^3 + 126x^4y^5 - 36x^2y^7 + y^9) + a_6(9xy^8 - 84x^3y^6 + 126x^5y^4 - 36x^7y^2 + x^9)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(9xy^8 - 84x^3y^6 + 126x^5y^4 - 36x^7y^2 + x^9) + a_6(-9x^8y + 84x^6y^3 - 126x^4y^5 + 36x^2y^7 - y^9)$$

(2) $q = 9$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(8x^7y - 56x^5y^3 + 56x^3y^5 - 8xy^7) + a_6(x^8 - 28x^6y^2 + 70x^4y^4 - 28x^2y^6 + y^8)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(x^8 - 28x^6y^2 + 70x^4y^4 - 28x^2y^6 + y^8) + a_6(8xy^7 - 56x^3y^5 + 56x^5y^3 - 8x^7y)$$

(3) $q = 8$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(21x^2y^5 - 35x^4y^3 + 7x^6y - y^7) + a_6(-7xy^6 + 35x^3y^4 - 21x^5y^2 + x^7) + b_7(54x^8y - 168x^6y^3 - 252x^4y^5 + 216x^2y^7 - 10y^9) + a_7(-8x^9 + 144x^7y^2 - 336x^5y^4 + 72xy^8)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6) + a_6(-7x^6y + 35x^4y^3 - 21x^2y^5 + y^7) + b_7(10x^9 - 216x^7y^2 + 252x^5y^4 + 168x^3y^6 - 54xy^8) + a_7(-8y^9 + 144x^2y^7 - 336x^4y^5 + 72x^6y^3)$$

(4) $q = 7$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(40x^7y - 56x^5y^3 - 168x^3y^5 + 56xy^7) + a_6(-7x^8 + 84x^6y^2 + 70x^4y^4 - 140x^2y^6 + 9y^8) + b_7(6x^5y - 20x^3y^3 + 6xy^5) + a_7(x^6 - 15x^4y^2 + 15x^2y^4 - y^6)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(9x^8 - 140x^6y^2 + 70x^4y^4 + 84x^2y^6 - 7y^8) + a_6(56x^7y - 168x^5y^3 - 56x^3y^5 + 40xy^7) + b_7(x^6 - 15x^4y^2 + 15x^2y^4 - y^6) + a_7(-6x^5y + 20x^3y^3 - 6xy^5)$$

(5) $q = 6$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(8y^7 - 84x^2y^5 + 28x^6y) + a_6(-6x^7 + 42x^5y^2 + 70x^3y^4 - 42xy^6) + b_7(13x^8y + 28x^6y^3 - 42x^4y^5 - 52x^2y^7 + 5y^9) + a_7(-3x^9 + 12x^7y^2 + 70x^5y^4 + 28x^3y^6 - 27xy^7) + b_8(y^5 - 10x^2y^3 + 5x^4y) + a_8(x^5 - 10x^3y^2 + 5xy^4)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(8x^7 - 84x^5y^2 + 28xy^6) + a_6(6y^7 - 42x^2y^5 - 70x^4y^3 + 42x^6y) + b_7(5x^9 - 52x^7y^2 - 42x^5y^4 + 28x^3y^6 + 13xy^8) + a_7(3y^9 - 12x^2y^7 - 70x^4y^5 - 28x^6y^3 + 27x^8y) +$$

$$b_8(x^5 - 10x^3y^2 + 5xy^4) + a_8(-y^5 - 5x^4y + 10x^2y^3)$$

$$(6) \quad q = 5$$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(20x^3y^3 + 18x^5y - 30xy^5) + a_6(-5x^6 + 15x^4y^2 + 45x^2y^4 - 7y^6) + b_7(8x^7y + 28x^5y^3 - 20xy^7) + a_7(-5/2x^8 + 35x^4y^4 + 28x^2y^6 - 9/2y^8) + b_8(4x^3y - 4xy^3) + a_8(x^4 - 6x^2y^2 + y^4) + b_9(9x^8y - 84x^6y^3 + 126x^4y^5 - 36x^2y^7 + y^9) + a_9(x^9 - 36x^7y^2 + 126x^5y^4 - 84x^3y^6 + 9xy^8)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(7x^6 - 45x^4y^2 - 15x^2y^4 + 5y^6) + a_6(30x^5y - 20x^3y^3 - 18xy^5) + b_7(9/2x^8 - 28x^6y^2 - 35x^4y^4 + 5/2y^8) + a_7(20x^7y - 28x^5y^3 - 8xy^7) + b_8(x^4 - 6x^2y^2 + y^4) + a_8(-4x^3y + 4xy^3) + b_9(x^9 - 36x^7y^2 + 125x^5y^4 - 84x^3y^6 + 9xy^8) + a_9(-9x^8y + 84x^6y^3 - 126x^4y^5 + 36x^2y^7 - y^9)$$

$$(7) \quad q = 4$$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(10x^4y + 20x^2y^3 - 6y^5) + a_6(-4x^5 + 20xy^4) + b_7(4x^6y + 20x^4y^3 + 12x^2y^5 - 4y^7) + a_7(-6x^5y^2 + 10x^3y^4 + 14xy^6 - 2x^7) + b_8(2x^8y + 56/3x^6y^3 + 28x^4y^5 + 8x^2y^7 - 10/3y^9) + a_8(-4/3x^9 - 8x^7y^2 + 56/3x^3y^6 + 12xy^8) + b_9(54x^8y - 168x^6y^3 - 252x^4y^5 - 216x^2y^7 - 10b_9y^9) + a_9(-8x^9 + 144x^7y^2 - 336x^5y^4 + 72xy^8) + b_{10}(3x^2y - y^3) + a_{10}(x^3 - 3xy^2) + b_{11}(7x^6y - 35x^4y^3 + 21x^2y^5 - y^7) + a_{11}(x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(6x^5 - 20x^3y^2 - 10xy^4) + a_6(20x^4y - 4y^5) + b_7(4x^7 - 12x^5y^2 - 20x^3y^4 - 4xy^6) + a_7(14x^6y + 10x^4y^3 - 6x^2y^5 - 2y^7) + b_8(10/3x^9 - 28x^5y^4 - 56/3x^3y^6 - 2xy^8 - 8x^7y^2) + a_8(12x^8y + 56/3x^6y^3 - 8x^2y^7 - 4/3y^9) + b_9(10x^9 - 216x^7y^2 + 252x^5y^4 + 168x^3y^6 - 54xy^9) + a_9(72x^8y - 336x^6y^3 + 144x^2y^7 - 8y^9) + b_{10}(x^3 - 3xy^2) + a_{10}(-3x^2y + y^3) + b_{11}(x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6) + a_{11}(-7x^6y + 35x^4y - 21x^2y^5 + y^7)$$

$$(8) \quad q = 3$$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(4x^3y + 12xy^3) + a_6(-3x^4 - 6x^2y^2 + 5y^4) + b_7(x^5y + 10x^3y^3 + 9xy^5) + a_7(-3/2x^6 - 15/2x^4y^2 - 5/2x^2y^4 + 7/2y^6) + b_8(8x^5y^3 + 16x^3y^5 + 8xy^7) + a_8(-x^8 - 8x^6y^2 + 10x^4y^4 + 3y^8) + b_9(28x^6y - 84x^2y^5 + 8y^7) + a_9(-6x^7 + 42x^5y^2 + 70x^3y^4 - 42xy^6) + b_{10}(13x^8y + 28x^6y^3 - 42x^4y^5 - 52x^2y^7 + 5y^9) + a_{10}(-3x^9 + 12x^7y^2 + 70x^5y^4 + 28x^3y^6 - 27xy^8) + 2b_{11}xy + a_{11}(x^2 - y^2) + b_{12}(5x^4y - 10x^2y^3 + y^5) + a_{12}(x^5 - 10x^3y^2 + 5xy^4) + b_{13}(8x^7y - 56x^5y^3 + 56x^3y^5 - 8xy^7) + a_{13}(x^8 - 28x^6y^2 + 70x^4y^4 - 28x^2y^6 + y^8)$$

$$\frac{dy}{dt} = x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + b_6(5x^4 - 6x^2y^2 - 3y^4) + a_6(12x^3y + 4xy^3) + b_7(7/2x^6 - 5/2x^4y^2 - 15/2x^2y^4 - 3/2y^6) + a_7(9x^5y + 10x^3y^3 + xy^5) + b_8(3x^8 - 10x^4y^4 - 8x^2y^6 - y^8) + a_8(8x^7y + 16x^5y^3 + 8x^3y^5) + b_9(8x^7 - 84x^5y^2 + 28xy^6) + a_9(42x^6y - 70x^4y^3 - 42x^2y^5 + 6y^7) + b_{10}(5x^9 - 52x^7y^2 - 42x^5y^4 + 28x^3y^6 + 13xy^8) + a_{10}(27x^8y - 28x^6y^3 - 70x^4y^5 - 12x^2y^7 + 3y^9) + b_{11}(x^2 - y^2) - 2a_{11}xy - b_{12}(x^5 - 10x^3y^2 + 5xy^4) + a_{12}(-5x^4y + 10x^2y^3 - y^5) + b_{13}(x^8 - 28x^6y^2 + 70x^4y^4 - 28x^2y^6 + y^8) + a_{13}(-8x^7y + 56x^5y^3 - 56x^3y^5 + 8xy^7)$$

$$(9) \quad q = 2$$

$$\frac{dx}{dt} = -y[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + 4b_6y^3 + a_6(-2x^3 - 6xy^2) + b_7(-x^4y + 2x^2y^3 + 3y^5) + a_7(-x^5 - 6x^3y^2 - 5xy^4) + b_8(-4/3x^6y + 4x^2y^5 + 8/3y^7) + a_8(-2/3x^7 - 6x^5y^2 - 10x^3y^4 - 14/3xy^6) + b_9(-3/2x^8y - 2x^6y^3 + 3x^4y^5 + 6x^2y^7 + 5/2y^9) + a_9(-1/2x^9 - 6x^7y^2 - 15x^5y^4 - 14x^3y^6 - 9/2xy^8) + b_{10}(10x^4y + 20x^2y^3 - 6y^5) + a_{10}(-4x^5 + 20xy^4) + b_{11}(4x^6y + 20x^4y^3 + 12x^2y^5 - 4y^7) + a_{11}(-2x^7 - 6x^5y^2 + 10x^3y^4 + 14xy^6) + b_{12}(2x^8y + 56/3x^6y^3 + 28x^4y^5 + 8x^2y^7 - 10/3y^9) + a_{12}(-4/3x^9 - 8x^7y^2 + 56/3x^3y^6 + 12xy^8) + b_{13}(28x^6y - 84x^2y^5 + 8y^7) +$$

$$\begin{aligned}
& a_{13}(-6x^7 + 42x^5y^2 + 70x^3y^4 - 42xy^6) + b_{14}(13x^8y + 28x^6y^3 - 42x^4y^5 - 52x^2y^7 + 5y^9) + a_{14}(-3x^9 + 12x^7y^2 + 70x^5y^4 \\
& + 28x^3y^6 - 27xy^8) + b_{15}(54x^8y - 168x^6y^3 - 252x^4y^5 + 216x^2y^7 - 10y^9) + a_{15}(-8x^9 + 144x^7y^2 - 336x^5y^4 + 72xy^8) \\
& + b_{16}y + a_{16}x + b_{17}(3x^2y - y^3) + a_{17}(x^3 - 3xy^2) + b_{18}(5x^4y - 10x^2y^3 + y^5) + a_{18}(x^5 - 10x^3y^2 + 5xy^4) + b_{19}(-y^7 \\
& + 7x^6y - 35x^4y^3 + 21x^2y^5) + a_{19}(x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6) + b_{20}(9x^8y - 84x^6y^3 + 126x^4y^5 - 36x^2y^7 + y^9) + \\
& a_{20}(x^9 - 36x^7y^2 + 126x^5y^4 - 84x^3y^6 + 9xy^8)
\end{aligned}$$

$$\begin{aligned}
\frac{dy}{dt} = & x[b_1 + b_2(x^2 + y^2) + b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + y^2)^4] + 4b_6x^3 + a_6(2y^3 + 6x^2y) + b_7(3x^5 + 2x^3y^2 - \\
& xy^4) + a_7(5x^4y + 6x^2y^3 + y^5) + b_8(8/3x^7 + 4x^5y^2 - 4/3xy^6) + a_8(14/3x^6y + 10x^4y^3 + 6x^2y^5 + 2/3y^7) + b_9(5/2x^9 \\
& + 6x^7y^2 + 3x^5y^4 - 2x^3y^6 - 3/2xy^8) + a_9(9/2x^8y + 14x^6y^3 + 15x^4y^5 + 6x^2y^7 + 1/2y^9) + b_{10}(6x^5 - 20x^3y^2 - 10xy^4) \\
& + a_{10}(20x^4y - 4y^5) + b_{11}(4x^7 - 12x^5y^2 - 20x^3y^4 - 4xy^6) + a_{11}(14x^6y + 10x^4y^3 - 6x^2y^5 - 2y^7) + b_{12}(10/3x^9 - 8x^7y^2 \\
& - 28x^5y^4 - 56/3x^3y^6 - 2xy^8) + a_{12}(12x^8y + 56/3x^6y^3 - 8x^4y^5 - 4/3y^9) + b_{13}(8x^7 - 84x^5y^2 + 28xy^6) + a_{13}(42x^6y - \\
& 70x^4y^3 - 42x^2y^5 + 6y^7) + b_{14}(5x^9 - 52x^7y^2 - 42x^5y^4 + 28x^3y^6 + 13xy^8) + a_{14}(27x^8y - 28x^6y^3 - 70x^4y^5 - 12x^2y^7 + \\
& 3y^9) + b_{15}(10x^9 - 216x^7y^2 + 252x^5y^4 + 168x^3y^6 - 54xy^8) + a_{15}(72x^8y - 336x^6y^3 + 144x^4y^7 - 8y^9) + b_{16}x - a_{16}y + \\
& b_{17}(x^3 - 3xy^2) + a_{17}(-3x^2y + y^3) + b_{18}(x^5 - 10x^3y^2 + 5xy^4) + a_{18}(-5x^4y + 10x^2y^3 - y^5) + b_{19}(x^7 - 21x^5y^2 + 35x^3y^4 \\
& - 7xy^6) + a_{19}(-7x^6y + 35x^4y^3 - 21x^2y^5 + y^7) + b_{20}(x^9 - 36x^7y^2 + 126x^5y^4 - 84x^3y^7 + 9xy^8) + a_{20}(-9x^8y + 84x^6y^3 \\
& - 126x^4y^5 + 36x^2y^7 - y^9)
\end{aligned}$$

证明 限于篇幅, 本文只对 $q = 10$ 时进行证明, 其它情况可类似得到.

由定理 1 知, 非平凡 z_{10} 等变平面九次 Hamilton 系统有如下形式:

$$\begin{aligned}
F(z, \bar{z}) = & (b_1 + b_2 |z|^2 + b_3 |z|^4 + \\
& b_4 |z|^6 + b_5 |z|^8) zi + A_6 \bar{z}^9
\end{aligned}$$

令 $z = x + yi$, $A_6 = a_6 + b_6 i$ (a_6, b_6 为实数) 则上式可写为:

$$\begin{aligned}
F(x, y) = F(z, \bar{z}) = & [b_1 + b_2(x^2 + y^2) + \\
& b_3(x^2 + y^2)^2 + b_4(x^2 + y^2)^3 + b_5(x^2 + \\
& y^2)^4](x + yi)i + (a_6 + b_6 i)(x - yi)^9 \quad (5)
\end{aligned}$$

又由 (2) 式可得:

$$\begin{cases} \dot{x} = (F(z, \bar{z}) + \overline{F(z, \bar{z})})/2 \\ \dot{y} = (F(z, \bar{z}) - \overline{F(z, \bar{z})})/2i \end{cases} \quad (6)$$

将 (5) 代入 (6) 中并进行化简即可得证.

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General Form of the Planar Nine Degree Equivariant Hamiltonian System

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Abstract: By using the theory of rotation symmetry group, the general form of nine-degree planar equivariant Hamiltonian vector field under Cartesian axis and complex axis is obtained.

Keywords: equivariant vector field; rotation symmetry group; general form