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A Novel Characterization of 2D Edges Based on Wavelet Transform*

YANG Li-hua, LIU Yi-shu, LI Feng
(Department of Scientific Computing and Computer Application, Zhongshan University, Guangzhou 510275, China)

Abstract: The characterization of curves in an image with wavelet transform is studied. Two novel wavelets are utilized to characterize and detect curves. Some significant characteristics of the local maximum moduli of the wavelet transform with respect to curves are presented. At last, the characteristics are applied to design a new algorithm to detect curves in an image.

Keywords wavelet transform; edge detection
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1 Introduction

Detection of edges is the main discipline in the areas of computer vision, image processing and pattern recognition. In the early years of edge detection, many detection operators including Roberts, Sobel, and Laplacian, were developed. These operators are very simple and can be implemented easily and effectively for most ideal edges. To improve the detection for noisy images, Marr suggested filtering the images with a Gaussian before detection. Instead of utilizing a given operator, Canny developed a computational approach to edge detection. It was suggested by Canny to detect edges by locating the local maxima of the output instead of the zero-crossings of its derivative. For detail references, see [1~3].

Due to the development of wavelet theory, Canny's approach is extended by Mallat and his coworkers[4]. It is proven that the maxima of the wavelet transform moduli detect the locations of irregular structures, and a numerical procedure to calculate the so called Lipschitz exponents is provided. Unfortunately, it is difficult to calculate the Lipschitz exponents exactly in practice. To develop new method for edge detection by utilizing wavelet theory, in [5], a significant property based on step edges has been established and then leads to provide a simple and direct strategy for detecting the step edges.

However, it is equally important to develop a method for detecting another important edges, the curves, i. e., Dirac-edges, in an image. In our latest paper [6], a very important characterization of curves by wavelet transform with respect to quadratic spline wavelet was provided. Three significant characteristics of the local maximum moduli of the wavelet transform with respect to the Dirac-edges are presented, namely: ① slope invariant; ② grey-level invariant; and ③ width light-dependent. It is interesting whether we can select better wavelets than quadratic spline so that the "width light-dependent" can be improved to "width invariant" without losing the "slope invariant" and "grey-level invariant". In this paper, such wavelets are presented. Due to the improvement, the detection of curves can be detected more exactly.

The paper is organized as follows: Section 2 is a brief review of the scale wavelet transform, then two novel wavelets are presented. In Section 3, a characterization of a straight line in an image by wavelet transforms is developed. At last, an algorithm to detect curves in an image based on the characterization above is designed and an experiment is carried out in Section 4.

2 Scale Wavelet Transform and the Construction of Two New Wavelets

Let $L^2(\mathbf{R}^2)$ be the Hilbert space of all the square-integrable 2-D functions on plane $\mathbf{R}^2$, $\psi \in L^2(\mathbf{R}^2)$ is called a wavelet function, if

$$\int_{\mathbf{R}}\int_{\mathbf{R}} \psi(x, y)\mathrm{d}x\mathrm{d}y = 0 \tag{1}$$

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收稿日期:2001-01-12;作者简介:杨力华(1962-),男,教授.

For $f \in L^2(\mathbf{R}^2)$ and scale $s > 0$, the scale wavelet transform of $f(x, y)$ is definet by

$$W_s^{\psi} f(x, y) := \int_{\mathbf{R}}\int_{\mathbf{R}} f(u, v) \frac{1}{s^2}\psi\left(\frac{x-u}{s}, \frac{y-v}{s}\right) \mathrm{d}u\mathrm{d}v \quad (2)$$

Curves are transient components with high frequencies in an image. They are highly localized in spatial positions, which makes their being analyzed and processed by the classical Fourier transforms not effective. To combat such a deficiency, wavelet analysis can be utilized.

However, unlike Fourier transform, there exist many different wavelets that can be utilized. Therefore, it is very important to select one as "good" as possible according to one's practical applications.

In this paper, two new wavelets are constructed as follows: Let $\phi(x)$ be an even function. On $[0, \infty)$, it is defined by

$$\phi(x) := \begin{cases} -\dfrac{2}{\pi}\displaystyle\int_x^1 \dfrac{\cos(2\pi t) - 1}{\sqrt{t^2 - x^2}}\mathrm{d}t & x \in [0,1) \\ 0 & x \geq 1 \end{cases} \quad (3)$$

The two new wavelets utilized in this paper are defined by

$$\begin{aligned} \psi^1(x, y) &:= \frac{\partial}{\partial x}\phi(\sqrt{x^2 + y^2}) \\ \psi^2(x, y) &:= \frac{\partial}{\partial y}\phi(\sqrt{x^2 + y^2}) \end{aligned} \quad (4)$$

and the corresponding wavelet transform modulus is defined by

$$M_s f(x, y) := \sqrt{|W_s^{\psi^1} f(s, y)|^2 + |W_s^{\psi^2} f(s, y)|^2} \quad (5)$$

Figure 1 is the graphic description of $\psi^1(x, y)$ (left) and $\psi^2(x, y)$ (right).

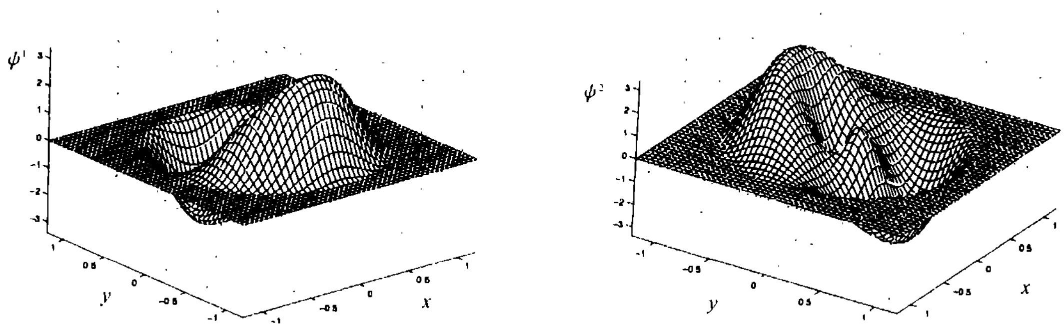

Fig. 1　The graphical descriptions of function $\psi^1(x, y)$ (left) and $\psi^2(x, y)$ (right)

3　Invariant Characterization of Curves through Wavelet Transform

In this section, three significant characteristics of the local maximum moduli of the wavelet transform with respect to Dirac-structure edges in images will be presented. Namely: ① Grey-level invariant: the local maximum moduli of the wavelet transforms with respect to a Dirac-structure edge takes place at the same points when the images with different grey-levels are to be processed; ② Slope invariant: the local maximum moduli of the wavelet transform of a Dirac-structure edge is independent of the slope of the edge; ③ Width invariant: for various widths of the Dirac-structure edges in an image, the location of maximum moduli of the wavelet transform does not vary under certain circumscription.

Let the parameter equation of a curve l_d with width d be $l_d: (u, v) = (u(t), v(t)), t \in [a, b]$. Then the equations of the curves parallel to l_d can be written as follows:

$$l_\lambda: \begin{cases} u_\lambda(t) = u(t) + \dfrac{\lambda v'(t)}{\sqrt{n'(t)^2 + v'(t)^2}} \\ v_\lambda(t) = v(t) - \dfrac{\lambda u'(t)}{\sqrt{n'(t)^2 + v'(t)^2}} \end{cases}, t \in [a, b]$$

where $|\lambda|$ is the distance between l_λ and the center line of l_d, and the sign of λ determines at which side of l_d l_λ locates. An image including only one curve l_d can be written as $f_{l_d}(x, y) = c_f \chi_{D_{l_d}}(x, y)$, where c_f is the grey level of the curve and $\chi_{D_{l-d}}(x, y)$ denotes the characteristic function of the area D_{l_d} which is defined by $D_{l_d} := \{(u_\lambda(t), v_\lambda(t)) \mid t \in [a, b], \lambda \in [-d/2, d/2]\}$.

We suppose l_d is a straight line for the simplicity of mathematical analysis. Let the parameter equation of the center line l is: $(u,v)=(u(t)+k_1(t-t_0),v(t)+k_2(t-t_0)),t\in[a,b]$, where k_1, k_2 denote the slope of l satisfying $k_1^2+k_2^2=1$, then for the wavelets defined by Eq. (4), we have

$$W_s^{\psi^i}f_{l_d}(x,y)=-c_f\int_a^b \mathrm{d}t\int_{-d/2}^{d/2}\psi_s^i(x-u(t_0)-k_1(t-t_0)-\lambda k_2,y-v(t_0)-k_2(t-t_0)+\lambda k_1)\mathrm{d}\lambda$$

For any $(u(t_0),v(t_0))\in l$, we consider the points in the normal line:

$$\begin{cases}x_\rho=u(t_0)+k_2\rho\\ y_\rho=v(t_0)+k_1\rho\end{cases}$$

where $|\rho|$ is the distance between (x_ρ,y_ρ) and $(u(t_0),v(t_0))$. Then it can be calculated that

$$W_s^{\psi^i}f_{l_d}(x,y)=-c_f\int_a^b \mathrm{d}t\int_{-d/2}^{d/2}\psi_s^i(k_2\rho-k_1(t-t_0)-\lambda k_2,-k_1\rho-k_2(t-t_0)+\lambda k_1)\mathrm{d}\lambda=$$
$$-c_f\int_{(t_0-b)/s}^{(t_0-a)/s}\mathrm{d}t\int_{(\rho-d/2)/s}^{(\rho+d/2)/s}\psi^i(k_1t+k_2\lambda,k_2t-k_1\lambda)\mathrm{d}\lambda\quad(i=1,2)$$

Similar to the discussion in [6], the "Grey-Level Invariant" and "Slope Invariant" can be easily proven. In the following, we focus on the proof of the "Width Invariant". That means parameter ρ introduced above retains the same value when $\rho\geq d$.

Now that $\phi(r)$ is compactly supported and the support is very short, for the point t_0 far away from a and b we can yield that

$$M_sf_{l_d}(x_\rho,y_\rho)=2\left|c_f\int_0^\infty\left[\phi\left(\sqrt{t^2+\left(\frac{\rho+d/2}{s}\right)^2}\right)-\phi\left(\sqrt{t^2+\left(\frac{\rho-d/2}{s}\right)^2}\right)\right]\mathrm{d}t\right|$$

To find ρ such that $M_sf_{l_d}(x_\rho,y_\rho)$ reaches the local maximum, we consider its derivative on ρ:

$$\frac{\mathrm{d}}{\mathrm{d}\rho}M_sf_{ld}(x_\rho,y_\rho)=-2|c_f|\int_0^\infty\left[\phi'\left(\sqrt{t^2+\left(\frac{\rho+d/2}{s}\right)^2}\right)\frac{\left(\frac{\rho+d/2}{s}\right)\frac{1}{s}}{\sqrt{t^2+\left(\rho+\frac{d/2}{s}\right)^2}}-\phi'\left(\sqrt{t^2+\left(\frac{\rho-d/2}{s}\right)^2}\right)\frac{\left(\frac{\rho-d/2}{s}\right)\frac{1}{s}}{\sqrt{t^2+\left(\frac{\rho-d/2}{s}\right)^2}}\right]\mathrm{d}t=$$
$$-2|c_f|\frac{1}{s}\left[F\left(\frac{\rho}{s}+\frac{d}{2s}\right)-F\left(\frac{\rho}{s}-\frac{d}{2s}\right)\right]$$

where $F(x)$ is an odd function defined by

$$F(x):=x\int_0^\infty\frac{\phi'(\sqrt{t^2+x^2})}{\sqrt{t^2+x^2}}\mathrm{d}t=\begin{cases}\cos(2\pi x)-1 & x\in[0,1)\\ 0 & x\geq1\end{cases}\tag{6}$$

Let $\frac{\mathrm{d}}{\mathrm{d}\rho}|\nabla W_sf_{l_d}(x_\rho,y_\rho)|=0$

we get

$$F\left(\frac{\rho}{s}+\frac{d}{2s}\right)=F\left(\frac{\rho}{s}-\frac{d}{2s}\right)$$

whose solutions are as follows: $s\geq d$ and

$$\rho_1=-\frac{s}{2},\rho_2=\frac{s}{2}$$

which are independent of the width d. The so called "width invariant" has been proven.

4 Algorithm and Experiments of Detection of Curves

Based on the characterization of a straight line in an image developed in Section 3, an algorithm to detect straight lines in an image can be designed. The result is also valid for general curves since a short segment of a curve can be regarded as a straight line approximately. Our algorithm to detect curves in an image is designed as follows.

Algorithm Let $f(x,y)$ be an image containing curves. For a scale $s>0$,

Step 1 Calculate the local maxima f_{locmax} of $M_sf(x,y)$ and the gradient direction f_{gradient} with respect to the wavelets defined by Eq. (4)

Step 2 For each point (x,y) with local maximum, search for the point whose distance along the gradient direction from (x,y) is s. If it is still a point of local maxima, the center point is detected.

Step 3 The curves formed by all the points detected in the last step are what we need.

Figure 2 is an experiment of curve detection. The left is an original image. By using the algorithm described above, the two crossing straight lines are detected, which are shown on the right of Figure 2.

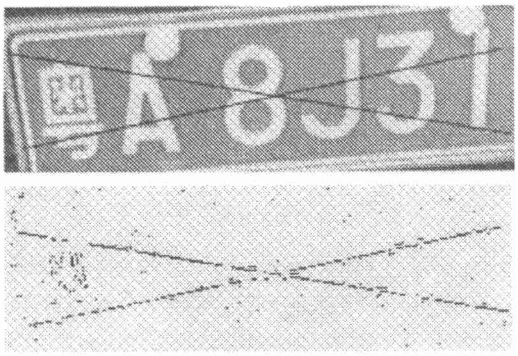

Fig.2　Detection of curves.
The scale of the wavelet transform is $s = 3$

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一种新的基于小波变换的 2 维边缘刻划特征

杨力华，刘亦书，李　峰
（中山大学科学计算与计算机应用系，广东 广州 510275）

摘　要：利用 2 个新的小波建立了图像中曲线的小波变换之局部极大模刻划特征，然后利用这些特征设计了检测图像中曲线的算法.

关键词：小波变换；边缘检测
中图分类号：O235

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The Homeomorphism Theorem of Bounded Semicharacters and Its Applications

HE Yuan-jiang
(Department of Statistics Science, Zhongshan University, Guangzhou 510275, China)

Abstract: Let $(S, +, e)$ be a commutative semigroup with neutral element e. A bounded semicharacter is a function $\rho: S \to [-1, 1]$ such that $\rho(e) = 1$ and $\rho(s+t) = \rho(s)\rho(t)$, $s, t \in S$. Let H be a set of some bounded semicharacters, $M(H)$ be the set of all finite Radon measures on H. Then holds the homeomorphism theorem: The mapping $\mu \to L_\mu := \int_H \rho(s)\mu(d\rho), s \in S$ is a homeomorphism from $M(H)$ to a subset of $\mathbf{R}^S$. Using this theorem, a new and simpler proof is given for the corresponding canonical propositions of the random measure on a locally compact space, without the second countability condition.

Keywords: semigroup; Radon measure; positive definite function; Laplace functional; random measure