

A General Form of Hybrid Equilibria in Job Market Signaling Games

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Abstract: The idea of bi hybrid equilibrium is presented. With this generalized definition of hybrid equilibrium, a general form of hybrid equilibria in job market signaling games is obtained. Both usual hybrid equilibria are special cases of this form.

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1 Frame of Job-Market Signaling Games

The enormous literature on signaling games begins with Spence' (1973) model^[1], which preceeded the widespread use of extensive form games to describe economic problems. Three kinds of perfect Bayesian equilibria^[2] are suggested to this job market signaling model: pooling, seperating and hybrid. The former two are relatively simple and exist in profution. The last one is somewhat complicated. This paper draws attention to hybrid equilibria.

The timing of Spence' s model is usually as follows^[2]:

(1) Nature determines a worker' s productive ability, which can be high (H) or low (L). The probability of $\eta = H$ is q .

(2) The worker learns his or her ability and then chooses a level of education, $e \geq 0$.

(3) Firms observe the worker' s education (but not the worker' s ability) and then simultaneously make wage offers to the worker.

(4) The worker accepts the higher of the two wage offers, flipping a coin in case of a tie. Let w denote the wage the worker accepts.

The payoffs are $w - c(\eta, e)$ to the worker, where $c(\eta, e)$ is the cost to a worker with ability of obtaining education level e ; $y(\eta, e) - w$ to the firm that employs the worker, where $y(\eta, e)$ is the output of a worker with ability η who has obtained education e ; and zero to the firm that does not employ the worker.

The assumption that competition exists among firms is required, which derives expected profits of work firm to zero. The case in which the worker' s ability is common knowledge among all the players is not

the core of job market signaling games. This paper focuses on the other case in which the worker' s ability is private information. This allows the possibility that a low ability worker could try to pretend to be a high ability one.

2 Hybrid Equilibrium and Its Generalized Definition

This section focuses on hybrid equilibria of Spence' s model.

2.1 Hybrid Equilibrium^[2]

Definition 1 A hybrid equilibrium is the equilibrium that one type of workers chooses one education level with certainty but the other type randomizes between pooling with the first type (by choosing the first type' s education level) and separating from the first type (by choosing a different education level).

A hybrid equilibrium in which the low ability worker randomizes is suggested by Gibbons^[2]:

Suppose the high ability worker chooses education level e_h (where the subscript "h" connotes "hybrid"), but the low ability worker randomizes between choosing e_h with probability π and choosing e_L with probability $1 - \pi$. Bayes' rule yields the rational firms' belief about the worker' s ability after observing e_h or e_L :

$$\mu(H | e_h) = q/[q + (1 - q)\pi]$$

$$\mu(H | e_L) = 0$$

For the low ability worker to be willing to radonmizes between separating at $e^*(L)$ and pooling at e_h the wage $w(e_h) = w_h$ must make worker indifferent between the two:

$$w^*(L) - c[L, e^*(L)] = w_h - c(L, e_h)$$

Here $(w^*(L), e^*(L))$ is the equilibrium on wage constraint to certain low ability worker. See Fig.

1, I_L and I_H are indifference curves of low ability worker and high ability worker respectively.

Given the firms belief and the zero profit assumption, w_h being an equilibrium wage for firms to pay implies

$$w_h = ry(H, e_h) + (1-r)y(L, e_h)$$

Here $r = q/[q + (1-q)\pi]$.

Gibbons suggests the firms' belief be that the worker has low ability if $e < e_h$ but otherwise has high ability with probability r and low ability with probability $1-r$, i. e.

$$\mu(H | e) = \begin{cases} 0 & e < e_h \\ \frac{q}{q + (1-q)\pi} & e \geq e_h \end{cases} \quad (1)$$

and the firms' strategy be

$$w(e) = \begin{cases} y(L, e) & e < e_h \\ ry(H, e) + (1-r)y(L, e) & e \geq e_h \end{cases} \quad (2)$$

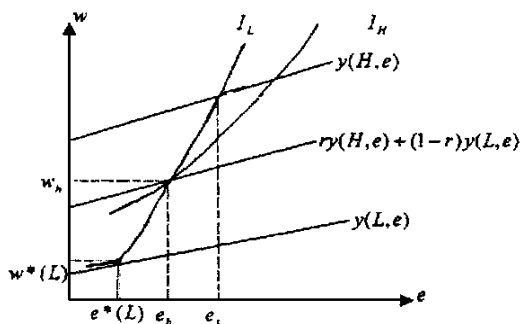


Fig 1 A hybrid equilibrium in which low-ability worker randomizes

2.2 Generalized Definition

Follow Gibbons, it is not difficult to get a hybrid equilibrium of the complementary case in which high ability worker randomizes. Equilibrium strategies suggested here are:

worker's strategy: low ability worker chooses education level e_h with certainty; high ability worker chooses e_h with probability π and e_H with probability $1-\pi$. Here the subscript "H" means "High". Therefore, from Bayes' rule, rational firms' belief is to be

$$\mu(H | e) = \begin{cases} 1 & e \geq e_H \\ \frac{q\pi}{q\pi + (1-q)} & e_h \leq e < e_H \\ 0 & e < e_h \end{cases} \quad (3)$$

and firms' strategy is

$$w(e) = \begin{cases} y(H, e) & e \geq e_H \\ ry(H, e) + (1-r)y(L, e) & e_h \leq e < e_H \\ y(L, e) & e < e_h \end{cases} \quad (4)$$

where $r = q\pi/[q\pi + (1-q)]$. See Fig. 2.

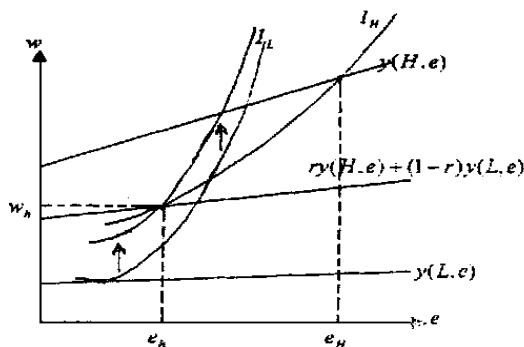


Fig 2 A hybrid equilibrium in which high-ability worker randomizes

Both cases of hybrid equilibria are based on the concept of "hybrid": one type chooses the hybrid level e_h with certainty and the other type randomizes between e_h and another education level. In equilibrium, firms receive only two signs, the chosen education levels. How about both types of workers randomize? This paper deals with this issue and originally presents the idea "bi hybrid equilibrium".

Definition 2 A bi hybrid equilibrium of job-market signaling games is the equilibrium case in which both types randomize with probability π_1 and π_2 respectively.

Follow Gibbons' suggestion, education levels are roughly divided into 3 classes: high school and under high school; college and master level; at least Ph D. In this sense of classification, bi hybrid equilibrium applies well to reality: not only some low ability employees select college or master level, but also some high ability employees do so.

3 Construction of Bi hybrid Equilibrium and Discussions

3.1 Construction of Equilibrium

To obtain the equilibrium of job market signaling games, worker's strategy, firms' belief and strategy are called for.

As to the requirements of bi hybrid, suppose low-ability worker randomizes between e_h with probability π_1 and $e^*(L)$ with probability $1-\pi_1$; High ability worker randomizes between e_h with probability π_2 and e_H with probability $1-\pi_2$. Here $e^*(L)$ is the optimal

choice for low ability worker restricted to the wage-offering line $w(e) = y(L, e)$, and subscript “H” connotes “High” (see Fig. 3).

Knowing this, the rational firms’ belief, appealing to Bayes’ rule, should be

$$\mu(H | e_h) = \begin{cases} 1 & e \geq e_H \\ \frac{q\pi_2}{q\pi_2 + (1-q)\pi_1} & e_h \leq e < e_H \\ 0 & e < e_h \end{cases} \quad (5)$$

Although both types of workers commonly randomize only to e_h according to their strategies, firms are allowed to believe that workers randomize to any level between e_h and e_H . It will be shown later that at equilibrium, workers commonly randomize to e_h only.

So, the firms’ strategy related to this belief is

$$w(e) = \begin{cases} y(H, e) & e \geq e_H \\ ry(H, e) + (1-r)y(L, e) & e_h \leq e < e_H \\ y(L, e) & e < e_h \end{cases} \quad (6)$$

where $r = q\pi_2 / [q\pi_2 + (1-q)\pi_1]$.

The broken wage offering line is illustrated in Fig. 3.

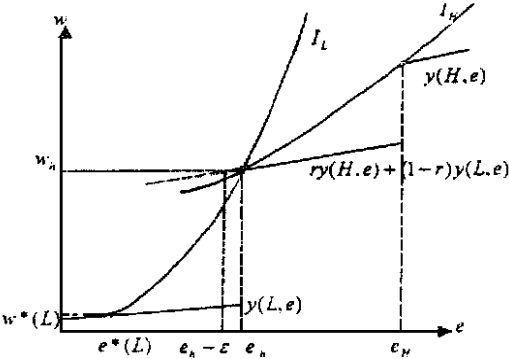


Fig. 3 Bi-hybrid equilibrium in which both types of workers randomize

The assumption of zero profit implies that the firms’ strategy is optimal to the worker’s strategy. It remains only to check that the worker’s strategy ($e(L) = e_h$ with probability π_1 , $e(L) = e^*(L)$ with probability $1 - \pi_1$; and $e(H) = e_h$ with probability π_2 , $e(H) = e_H$ with probability $1 - \pi_2$) is a best response to the firms’ strategy. See Fig. 3, for the low-

ability worker, the optimal $e < e_h$ is $e^*(L)$ and the optimal $e \geq e_h$ is e_h , while for the high ability worker, the optimal $e < e_H$ is e_h and the optimal $e \geq e_H$ is e_H . Thus workers never choose education levels rather than $e^*(L)$, e_h and e_H .

Now the strategies of worker and firms reach a bi-hybrid equilibrium. It is a perfect Bayesian equilibrium^[2].

3.2 Discussions

(1) A question may naturally be proposed that whether there exists a kind of “more general” hybrid equilibrium that workers randomize but do not have common education level, i.e. firms may receive 4 signs (education levels). It is not the case, because the low ability worker has no incentive to deviate from the high ability worker’s less level, e_h . Otherwise she or he will be distinguished and offered low wage.

(2) If firms ignore the certainty of high ability worker or set the critical level e_H high enough, then $\pi_2 = 1$ and equations (5), (6) become equations (1), (2) respectively. If firms ignore the certainty of low ability worker, then $\pi_1 = 1$, and equations (5), (6) are equations (3), (4) respectively.

(3) The bi hybrid equilibrium may generate a trivial case in which both types of workers choose e_h certainly. See Fig. 3, if e_h is set a bit less, to $e_h - \epsilon$, both types of workers have incentive to choose $e_h - \epsilon$ with certainty, abandoning alternative choices $e^*(L)$ and e_H .

This case of wholly hybrid equilibrium is beyond book [2].

Discussions (1), (2) and (3) imply that this bi-hybrid equilibrium is really a general form of hybrid equilibrium in job market signaling games.

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工作市场信号博弈混同均衡通式

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摘要: 提出双混同均衡的概念, 作为对混同均衡的推广. 基于该概念, 给出了工作市场信号博弈混同均衡的一般形式. 因而, 普通的混同均衡都是它的特例.

关键词: 博弈论; 工作市场信号博弈; 混同均衡