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# Self-similarity Property of the Output Process in ATM Networks*

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**Abstract:** A model of a single server with infinite buffer is established and it is shown that if the queue length has finite second-order moment, then the input process being strong asymptotically second-order self-similar (sas-s) is equivalent to the output process also bearing the sas-s property.

**Keywords:** ATM networks; self-similar; queueing analysis

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Several empirical studies[1,2] on the LAN, the VBR video traffic, the ISDN and other communication systems indicate that these traffic are self-similar in nature. In the light of these strong experimental evidence it is important to examine in more detail the possible implications that self-similar traffic may have on the design and performance of network systems. In particular, an important question is whether a server mechanism will change the self-similarity nature of the traffic. Vamvakos et al[3] consider a special case of a leaky bucket system with long-range dependent input traffic, and prove that the output (departure) process is also long-range dependent.

In this paper, we consider a model of a single server with infinite buffer, and prove that when the second-order moment of a queue length process is finite, the strong asymptotically second-order self-similarity (sas-s) properties of the input process and of the output process are equivalent, which means that the self-similarity property will not be removed or added by any server mechanism with finite second-order moment of queue length.

## 1 Definition of Self-similarity

We begin with the introduction of $X = \{X_t, t \in I_1\}$, a semi-infinite segment of a second-order-stationary real-number stochastic process of discrete argument (time) $t \in I_1 \triangleq \{1, 2 \cdots\}$. Denote $\mu = EX_t < \infty$ and $\sigma^2 = \mathrm{var} X_t < \infty$ are $X_t$'s mean and variance, respectively.

Denote the correlation coefficient of process $X$ by

$$r(k) \triangleq \frac{E(X_{t+k} - \mu)(X_t - \mu)}{\sigma^2} \qquad (1)$$

**Definition 1** A process $X$ is called *exactly second-order self-similar (es-s)* with Hurst parameter H if its correlation coefficient is

$$r(k) = \frac{1}{2}[(k+1)^{2-\beta} - 2k^{2-\beta} + (k-1)^{2-\beta}] \triangleq g(k), k \in I_1 \qquad (2)$$

**Definition 2** A second-order stationary process X is called long-range dependant process *(l-rd)* with Hurst parameter $H = 1 - \beta/2, 0 < \beta < 1$, if its correlation coefficient satisfies

$$r(k) \sim ck^{-\beta}, k \to \infty \qquad (3)$$

$X^{(m)} = \{X_t^{(m)}, t \in I_1\}$ is the averaged (over blocks of length $m$) process of $X$,

$$X_t^{(m)} = \frac{1}{m}(X_{tm-m+1} + \cdots + X_{tm}), m, t \in I_1 \qquad (4)$$

$V_m = \mathrm{Var} X_t^{(m)}$ and $r_m(k)$ are variance and correlation coefficient respectively.

**Definition 3** A second-order stationary process $X$ is called asymptotically second-order self-similar (as-s) with Hurst parameter $H = 1 - \beta/2, 0 < \beta < 1$, if

$$\lim_{m\to\infty} r_m(k) = g(k), k \in I_1 \qquad (5)$$

**Definition 4** A second-order stationary process X is called strong asymptotically second-order self-similar (sas-s) with Hurst parameter $H = 1 - \beta/2, 0 < \beta < 1$, if

$$V_m \sim dL(m) m^{-\beta}, m \to \infty \qquad (6)$$

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where $L(m)$ is a slowly varying function ($svf$, Refer to the Appendix E of [4]), $c$ is a constant, $0 < c < \infty$.

According to the Definitions above and the Theorem 2 of [5], we have the following relationships

$$\text{X is } es\text{-}s \Rightarrow \text{X is } lrd \Rightarrow$$
$$\text{X is } sas\text{-}s \Rightarrow \text{X is } as\text{-}s$$

## 2 The main result

We consider a single server queueing system with infinite buffer. For simplicity, suppose there are two classes of customers, and denote the input processes of the two classes of customers by $X$ and $Y$, such that $X$, and $Y$, respectively, are the corresponding output processes. $(X, X')$ is the input-output processes pair that we are going to study, and $(Y, Y')$ represents all other input-output processes pair. Denote $X^{(m)}, X'^{(m)}$ the averaged (over blocks of length $m$) processes of $X, X'$ respectively. We define $X$, the queue length process corresponding to the arrival process X as

$X_t$=the number of customers in queue from $X$ at time $t-$.

Assume that the average service rate is $\gamma$, and input processes $X$, $Y$ are stationary and ergodic with arrival rate of $\alpha$, $\beta$ which satisfy the stability condition of $\alpha+\beta<\gamma$. According to the results in [3, 5], we may consider a stationary regime in which the output process $X'$, the queue length process, $X$, is stationary and ergodic, and has finite second order moment, (i.e. $EX_t^2 < \infty$). Let $r'_m(k)$ be the correlation coefficient of $X'^{(m)}$.

**Theorem** In stationary regime, if $X_t$ has a finite second order moment, then the followings are equivalent:

(1) $X$ is $sas$-$s$ with Hurst parameter H (i.e. $V_m \sim cL(m)m^{-\beta}, m \to \infty$).

(2) $X'$ is $sas$-$s$ with Hurst parameter H (i.e. $V'_m \sim cL(m)m^{-\beta}, m \to \infty$).

where $L(m)$ is a slowly varying function ($svf$), $H = 1-\beta/2, 0<\beta<1$, and $c$ is a constant.

**Proof:** First of all, we show that

$$X_t^{(m)} - X_t'^{(m)} = (X_{tm} - X_{(t-1),m})/m \quad (7)$$

Denote by $A$ and $C$ the time points $(t-1)m$, and $tm$, respectively. $B$ and $D$ are the corresponding virtue output time points of a customer input at time $(t-1)m^+$, $tm^+$ under a FIFO regime. Note that it is uncertain that there is a customer input at this time, and the service is not necessary FIFO which will not effect the queue length and output process. We denote the numbers of output customers of input process $X$ among the time intervals $AB$, $AC$, $BD$, $CD$ by $\#(AB)$, $\#(AC)$, $\#(BD)$, $\#(CD)$, repectivily. Therefore, we have

$$X_t^{(m)} = \#(BD)/m, X_t'^{(m)} = \#(AC)/m,$$
$$X_{tm} = \#(CD), X_{(t-1)m} = \#(AB) \quad (8)$$

then

$$X^{(m)} - X_t'^{(m)} = |\#(BD) - \#(AC)|/m,$$
$$X_{tm} - X_{(t-1)m} = |\#(CD) - \#(AB)| \quad (9)$$

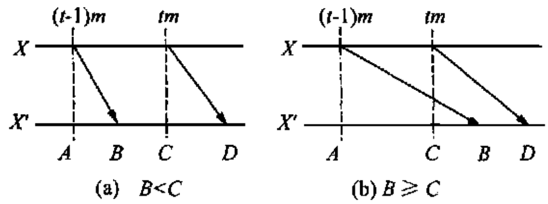


Fig. 1 The input and output points of $X$

Note that if $B < C$ (as in Fig. 1a) then

$$\#(BD) = \#(CD) + \#(BC),$$
$$\#(AC) = \#(AB) + \#(BC)$$

On the other hand, if $B \geqslant C$ (as in Fig. 1b) then

$$\#(BD) = \#(CD) - \#(CD),$$
$$\#(AC) = \#(AB) - \#(CB)$$

In both cases, it is true that

$$\#(BD) - \#(AC) = \#(CD) - \#(AB) \quad (10)$$

That is $X_t^{(m)} - X_t'^{(m)} = (X_{tm} - X_{(t-1)m})/m$.

Next we will prove an inequality

$$\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) - \text{var}^{1/2}(V_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}) \leqslant \text{var}^{1/2}(X_t'^{(m)}/V_m^{1/2}) \leqslant \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) + \text{var}^{1/2}(V_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}) \quad (11)$$

This inequality is equivalent to

$$|\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) - \text{var}^{1/2}((X_t'^{(m)}/V_m^{1/2})| \leqslant \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}) \quad (12)$$

and (12) are squared in both sides

$$\text{var}(X_t^{(m)}/V_m^{1/2}) + \text{var}(X_t'^{(m)}/V_m^{1/2}) - 2\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2})\text{var}^{1/2}(X_t'^{(m)}/V_m^{1/2}) \leqslant \text{var}(X_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}) \quad (13)$$

Using $\text{var}(X-Y) = \text{var}X + \text{var}Y - 2\text{cov}(X, Y)$, we can translate (13) into

$$\text{cov}(X_t^{(m)}/V_m^{1/2}), X_t'^{(m)}/V_m^{1/2}) \leqslant \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2})\text{var}^{1/2}(X_t'^{(m)}/V_m^{1/2}) \quad (14)$$

That is right because

$$\frac{\text{cov}(X_t^{(m)}/V_m^{1/2}), X_t'^{(m)}/V_m^{1/2})}{\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2})\text{var}^{1/2}(X_t'^{(m)}/V_m^{1/2})} \leqslant 1 \quad (15)$$

So, we have proved (11).

Now we will prove that if $V_m \sim cL(m)m^{-\beta}$, $m$

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$\to \infty$, then $V'_m \sim cL(m)m^{-\beta}, m \to \infty$.

By (9), we obtain

$$X_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2} = (X_{tm} - X_{(t-1)m})/mV_m^{1/2} \quad (16)$$

Since $V_m \sim cL(m)m^{-\beta}$, we have

$$mV_m^{1/2} \sim m\{cL(m)m^{-\beta}\}^{1/2} = \{cL(m)m^{2-\beta}\}^{1/2} \to \infty, m \to \infty \quad (17)$$

By stationary and $X_t$ has a finite second order moment, we have

$$\mathrm{var}(X_{tm}) = \mathrm{var}(X_{(t-1)m}) = \mathrm{var}(X_1) < \infty \quad (18)$$

Using (17) and (18), we can get the limit of the variance of the right hand of (16)

$$\lim_{m\to\infty}\mathrm{var}[(X_{tm} - X_{(t-1)m})/mV_m^{1/2}] = 0 \quad (19)$$

So, the variance of the left hand side of (16) has limit

$$\lim_{m\to\infty}\mathrm{var}[X_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}] = 0 \quad (20)$$

Since

$$\mathrm{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) = \mathrm{var}^{1/2}(X_t^{(m)})/V_m^{1/2} = 1 \quad (21)$$

(11) can be written as

$$1 - \mathrm{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}) \leqslant \mathrm{var}^{1/2}(X_t'^{(m)}/V_m^{1/2}) \leqslant 1 + \mathrm{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X_t'^{(m)}/V_m^{1/2}) \quad (22)$$

Let $m\to\infty$, we get that

$$\lim_{m\to\infty}\mathrm{var}(X_t'^{(m)}/V_m^{1/2}) = 1 \quad (23)$$

By the definition of $V'_m$, we have

$$\lim_{m\to\infty} V'_m/V_m = 1 \quad (24)$$

that is $$V'_m \sim cL(m)m^{-\beta}, m \to \infty \quad (25)$$

Reversely, by analogous arguments, we can prove that $X$ is also *sas-s*, if $X'$ is *sas-s*.

# 3 Application

This result provides strong evidence to support the empirical observation that there is self-similarity (or long-range dependence) in traffic carried by various networks. Indeed, what it has demonstrated is that this self-similarity property cannot be removed by the kinds of flow control schemes that retains finite second-order moment of queue length.

The result can be applied to network traffic control for statistical end-to-end delay guarantee on the networks. In [6], a statistical delay bound on an single ATM switch with self-similar Input Traffic has been given, according our result, the output process retains the same self-similarity property of the input process. Therefore, we can apply the same statistical delay bound on the subsequent ATM switches for this connection. The advantage is that we only need to calcuitches along the same connection. Furthermore, given the method for determinating the worst case cell delay for an ATM switch with self-similar input traffic, with this proof, we can determine the end-to-end delay for such real-time communications in an ATM network by summing the cell delay experienced by each of the ATM switch in the connection.

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# ATM 网络中输出过程的自相似特性


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摘 要: 建立一个有无限缓冲区的单服务器模型, 证明了在队长有有限二阶矩的条件下, 输出过程具强渐近二阶自相似性质与输出过程具强渐近二阶自相似性质等价.



关键词: ATM 网络; 自相似; 排队分析


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