Characterization of 1-Extendable Bipartite Graphs*

LOU Ding-jun, WANG Wei
(Department of Computer Science, Sun Yat-sen University, Guangzhou 510275, China)

Abstrace: It is shown that let G be a bipartite graph withbipartition (X, Y) and with a perfect matching M, then G is 1-extendable if and only if G has an ear decomposition $G = e + P_1 + P_2 + \cdots + P_r$ such that $e \in M$ and each P_i is an M-alternating path starting and ending with edges in $E(G) \setminus M$. Then a polynomial time algorithm is provided to determine 1-extendable bipartite graph by finding the ear decomposition.

Keywords: bipartite graph; 1-extendable graph; ear decomposition; M-alternating path
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1 Introduction and terminology

This paper only discusses bipartite graphs. All graphs are finite, connected and simple. For the terminology and notation not defined in this paper, the reader is referred to [1] and [2].

Let G be a bipartite graph with bipartition (X, Y). Let M be a matching of G and $\upsilon \in V(G)$. If υ is incident with an edge in M, then υ is said to be M-saturated, otherwise υ is said to be M-unsaturated. If every vertex of G is M-saturated, then M is a perfect matching.

The graph G is said to be 1-extendable if every edge in G lies in a perfect matching. Let e be an edge. Join its ends by a path P_1 of odd length (the so called "first ear"). The reader may now proceed inductively to build a sequence of bipartite graphs as follows. If $G_{r-1} = e + P_1 + \cdots + P_{r-1}$ has already been constructed, add an rth ear P_r by joining any two vertices in different colour classes of G_{r-1} by an odd path (i.e. P_r) having no other vertex in common with G_{r-1}. The decomposition $G_r = e + P_1 + \cdots + P_r$ will be called an ear decomposition of G_r.

The idea of Ear Decomposition of 1-extendable bipartite graphs was first introduced by Hetyei[3], and it was developed by Lovász and Plummer[4]. The concept of 1-extendable graph is later generalized to n-extendable graph (see [5] and [6]).

This paper shows that, for any perfect matching M of G, if G is 1-extendable bipartite, then not only G has an ear decomposition $G = e + P_1 + \cdots + P_r$, but also $e \in M$ and each P_i is an M-alternating path starting and ending with edges in $E(G) \setminus M$. In the last section, an efficient algorithm is given to determine 1-extendable bipartite graph by finding such an ear decomposition.

2 A necessary and sufficient condition for 1-extendable bipartite graphs

In the following, the authors shall give a necessary and sufficient condition for 1-extendable bipartite graphs using the ear decomposition such that each ear is an M-alternating path starting and ending with edges in $E(G) \setminus M$.

Theorem 1 Let G be a bipartite graph with bipartition (X, Y) and with a perfect matching M. Then G is 1-extendable if and only if G has an ear decomposition $G = e + P_1 + P_2 + \cdots + P_r$ such that $e \in M$ and each P_i is an M-alternating path starting and ending with edges in $E(G) \setminus M$.

Proof Necessity first. Suppose G is 1-extendable. Let $e = xy \in M$. Since $|V(G)| \geqslant 4$ and G is connected, either x or y is adjacent to another vertex, without loss of generality, assume $xz \in E(G)$ and $z \neq y$. Then $xz \in E(G) \setminus M$. By the 1-extendability of G, xz can be extended to a perfect matching M'. Then the component C of $G[M \Delta M']$ containing zxy is an M-alternating cycle on which the edge appear in M and M' alternately, where $M \Delta M'$ is symmetric difference of M and M'. Let $P_1 = C - xy$. Then P_1 is an M-alternating path starting and ending with edges in $E(G) \setminus M$. Now the authors proceed inductively. Assume the authors have constructed $G_{r-1} = e + P_1 + P_2 + \cdots + P_{r-1}$ such that each P_i is an M-alternating path starting and ending with

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作者简介：娄定俊（1962 年生），男，教授，博士生导师；E-mail：issldj@zsu.edu.cn

edges in $E(G)\setminus M$. Suppose $G\neq G_{r-1}$. Since G is connected, there is an edge $wz\notin E(G_{r-1})$ and $w\in V(G_{r-1})$. Since every vertex in G_{r-1} is incident with an edge in $E(G_{r-1})\cap M$, then $wz\notin M$. By the 1-extendability of G, wz can be extended to a perfect matching M'. Then the component C of $G[M\Delta M']$ containing $w'wz$ is an M-alternating cycle on which the edges appear in M and M' alternately, where $w'w\in E(G_{r-1})\cap M$. Let $P_r = wz\cdots u$ be the segment of C such that $V(P_r)\cap V(G_{r-1})=\{w, u\}$, then P_r is an M-alternating path starting and ending with edges in $E(G)\setminus M$. Then the authors get $G_r = e+P_1+P_2+\cdots+P_r$, continue this procedure until $G=G_r$ for some positive integer r. Then the authors get the ear decomposition $G=e+P_1+P_2+\cdots+P_r$ as required.

Now, the sufficiency. Suppose G has an ear decomposition $G=e+P_1+P_2+\cdots+P_r$, such that $e\in M$ and each P_i is an M-alternating path starting and ending with edges in $E(G)\setminus M$. Then each path P_i has odd length joining two vertices in different colour classes of G_{i-1} having no other vertex in common with G_{i-1}. By Theorem 4.1.6 in [2], G is 1-extendable. Hence the theorem is proved.

3 A polynomial time algorithm for the Ear Decomposition

This section gives a polynomial time algorithm to find an ear decomposition $G=e+P_1+P_2+\cdots+P_r$ such that $e\in M$ and each P_i is an M-alternating path starting and ending with edges in $E(G)\setminus M$. If such an ear decomposition exists, then it can be determined that G is 1-extendable and also the ear decomposition of G is obtained. Otherwise G is not 1-extendable.

Algorithm 1

(1) Use the algorithm in [7] to find a perfect matching M in G; // $O(|V|^{1/2}|E|)$

(2) If M does not exist, then RETURN (false); // $O(1)$ and G is not 1-extendable.

(3) Let $e\in M$ and let $G_0 := e$, $r:=0$; // $O(1)$

(4) While $G\neq G_r$ do // $O(|E|^2)$

BEGIN

(5) Take an edge wz such that $w\in V(G_r)$ and $wz\notin E(G_r)$; // $O(E)$

(6) Use Procedure 2 to find an M-alternating path P_{r+1} joining two vertices w and y in two different colour classes of G_r having no other vertex in common with G_r, such that P_{r+1} starts with the edge wz and ends with an edge in $E(G)\setminus M$; // $O(|E|)$

(7) If such a path P_{r+1} does not exist, then RETURN (false); // $O(1)$ and G is not 1-extendable

(8) Otherwise $G_{r+1} := G_r + P_{r+1}$; // $O(|V|)$

(9) $r := r+1$ // $O(1)$

END;

(10) RETURN (true) and $G := e+P_1+P_2+\cdots+P_r$; // $O(1)$ and G is 1-extendable

Procedure 2 builds an M-alternating tree T rooted at w starting with the edge wz using the edges not in $E(G_r)$.

The authors use the same idea for finding M-alternating path as in the algorithm to find a perfect matching in a bipartite graph. If T reaches a vertex y (possibly $y=z$) in $V(G_r)$, then the authors find a (w, y) M-alternating path P_{r+1} starting and ending with edges in $E(G)\setminus M$. If T does not reach any vertex y in $V(G_r)$ until T can not be extended further, then such an M-alternating path P_{r+1} does not exist. Procedure 2 uses breadth first search algorithm to build the M-alternating tree T to find the M-alternating path P_{r+1}, it takes $O(|E|)$ time.

Now, analyse the time complexity of Algorithm 1. The while loop of Step 4 repeats at most $O(|E|)$ times, since every time P_{r+1} is added to G_r and P_{r+1} has at least one edge for each r. Steps 5 and 6 take at most $O(|E|)$ time, and Step 8 takes at most $O(|V|)$ time. The whole loop of Steps 4—9 totally takes at most $O(|E|^2)$ time. So Algorithm 1 takes at most $O(|E|^2)$ time.

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馈电微带漏波天线此特性可用于实现频率固定下微带漏波天线主波束扫描，广泛地应用于移动通信以及雷达防撞系统，而且能应用于微带漏波天线阵的设计。

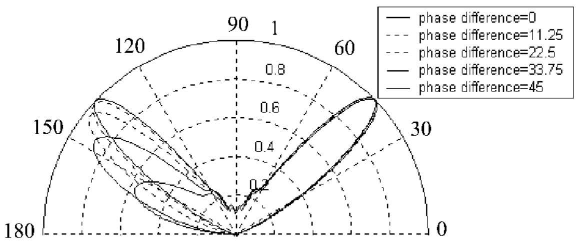

图 3 微带漏波天线两端馈电端口输入电流相位差为 0～45

Fig. 3 Pattern as the phase difference varies betwween 0～45°

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Simulation of a Microstrip Leaky-Wave Antenna with Two-Terminal Feeding

LI Yuan-xin, LONG Yun-liang

(Department of Electronics & Communication Engineering, Sun Yat-sen University, Guangzhou 510275, China)

Abstract: The theoretical analysis of a microstrip leaky-wave antenna fed by two terminals is presented. Besides the beam-switchable property, a new character of the beam scanning at the fixed-frequency of the antenna was discovered, which is believed to be useful in the application of mobile communications.

Key words: microstrip leaky-wave antenna; two-terminal feeding; phase difference

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1-可扩偶图的刻划

娄定俊，王 伟

(中山大学计算机科学系，广东 广州 510275)

摘 要：设 G 是一个具有二分类 (X, Y) 的偶图且 M 是 G 的一个完美对集。文章证明：G 是 1-可扩图当且仅当 G 有如下耳朵分解 $G = e + P_1 + P_2 + \cdots + P_r$ 使得 $e \in M$ 并且每个 P_i 是起始和终止边都在 $E(G) \setminus M$ 中的 M-交错路。文章还给出一个有效算法判定一个偶图是否 1-可扩图并找出该图的耳朵分解。

关键词：偶图；1-可扩图；耳朵分解；M-交错路

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