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带有负顾客的双端排队系统*

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摘 要：研究了有负顾客到达且系统空间有限的双端排队系统, 队列顾客端为批量到达, 到达间隔为一般分布, 而服务端为泊松到达。论文用补充变量法, 通过迭代给出了系统的稳态概率。最后对顾客单个到达且到达间隔为 Γ分布的情形给出了更明确的表达式。

关键词：双端队列；负顾客；批到达；被充变量法
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1 模型的描述

双端队列模型在日常生活中有许多背景, 例如, 机场出口的乘客与出租车公共等候空间。当乘客多于出租车时, 乘客形成一端队列, 而当出租出多于乘客时, 则出租车形成另一端队列。在排队过程中, 由于各种因素, 排队者也可能中途退出。

分别有不少文献研究双端排队系统[1-4]和带负顾客的排队系统[5-8], 但将它们综合在一起的研究还没有看到。本文就考虑这种带负顾客的双端排队系统。

假设顾客端(乘客) 正顾客批量到达, 到达时间间隔为一般分布, 同一批来到的顾客数X是一有限正整数值随机变量, 且各到达时刻的顾客数是相互独立的。服务端(出租车) 到达形成一强度为 λ 的泊松过程, 每辆出租车到达时, 若系统中有顾客等待, 则和在前端排队的顾客一起同时离开系统。若顾客端没有顾客, 则停留在系统中等待顾客的到来。顾客端与服务端的等待空间均有限, 分别为 N 和 M。当系统容量达到饱和时, 多余的顾客或服务员将离开系统。系统同时有负顾客以强度为 μ 的泊松过程到达, 负顾客到达系统后, 若发现系统中有正顾客, 则立刻随机地带走一个正顾客, 若系统中无正顾客则立刻自行消失。

设顾客端正顾客到达时间间隔的分布函数为 $F(x)$, 对应的密度函数和失效率函数分别为 $f(x)$ 和 $\eta(x)$, $\bar{F}(x) = 1 - F(x)$; 批量到达的顾客数的概率分布为 $g_k = P(X = k)(k = 1, \cdots, S, S = \min(M, N))$ 且有均值 EX。进一步分别记 $U(t)$ 和 $V(t)$ 为时刻 t 系统中的顾客数和出租车数, $N(t) = U(t) - V(t) = n, (n = -M, -M+1, \cdots, -1, 0, 1, 2, \cdots, N)$。则 $n > 0$ 表示时刻 t 系统中有 n 个顾客在等待服务, $n < 0$ 表示时刻 t 系统中有 $|n|$ 辆出租车在等待乘客, $n = 0$ 表示时刻 t 系统为空。引入补充变量 $R(t)$ 为时刻 t 与 t 前最后一批顾客到达时刻的时间间隔, 于是过程 $\{N(t), R(t), t \geqslant 0\}$ 形成一个马尔可夫过程。

2 系统的稳态方程

记

$$P_n(x, t)\mathrm{d}t = P\{N(t) = n, x < R(t) \leqslant x + \mathrm{d}t\},$$
$$n = -M, \cdots, -1, 0, 1, 2, \cdots, N$$

因为系统空间有限, 故系统必存在平稳状态。记

$$P_n(x) = \lim_{t\to\infty} P_n(x, t),$$
$$P_n(0) = \lim_{t\to\infty} P_n(0, t)$$

则系统的稳态微分方程组为

$$\frac{\partial P_{-M}(x)}{\partial x} + \eta(x) P_{-M}(x) = \lambda P_{-M+1}(x) \quad (1)$$

$$\frac{\partial P_n(x)}{\partial x} + [\lambda + \eta(x)] P_n(x) = \lambda P_{n+1}(x), \quad (-M+1 \leqslant n \leqslant -1) \quad (2)$$

$$\frac{\partial P_0(x)}{\partial x} + [\lambda + \eta(x)] P_0(x) = (\lambda + \mu) P_1(x) \quad (3)$$

$$\frac{\partial P_n(x)}{\partial x} + [\lambda + \mu + \eta(x)] P_n(x) =$$

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$$(\lambda+\mu)P_{n+1}(x),(1\leqslant n\leqslant N-1) \tag{4}$$

$$\frac{\partial P_N(x)}{\partial x}+[\lambda+\mu+\eta(x)]P_N(x)=0 \tag{5}$$

满足边界条件

$$P_{-M}(0)=0 \tag{6}$$

$$P_n(0)=\int_0^\infty \eta(x)\left[g_{n+M}P_{-M}(x)+\sum_{k=1}^{n+M+1}g_kP_{n-k}(x)\right]\mathrm{d}x,$$
$$(-M+1\leqslant n\leqslant -M+S) \tag{7}$$

$$P_n(0)=\int_0^\infty \eta(x)\sum_{k=1}^{S}g_kP_{n-k}(x)\,\mathrm{d}x,$$
$$(-M+S+1\leqslant n\leqslant N-1) \tag{8}$$

$$P_N(0)=\int_0^\infty \eta(x)\left[\sum_{k=1}^{S}h_kP_{N-k}(x)+P_N(x)\right]\mathrm{d}x \tag{9}$$

3　求解 $P_n(x)$

由方程组(1)~(5)易见，从(5)式出发进行迭代可求出 $P_n(x)$。首先由方程(4)~(5)得：

$$P_n(x)=\left[\sum_{r=0}^{N-n}\frac{(\lambda+\mu)^r}{r!}P_{n+r}(0)\,x^r\right]\cdot\exp\left\{-(\lambda+\mu)x-\int_0^x\eta(u)\,\mathrm{d}u\right\},(1\leqslant n\leqslant N)$$

用变量代换和分部积分以及迭代不难证明以下几个命题，在方程组的继续求解中将用到它们。

命题 1　设 $\gamma(r+1,x)=\int_0^x\exp(-t)\,t^r\mathrm{d}t$，$r=0,1,2\cdots$ 为不完全 Gamma 函数，则

$$\int_0^x\exp(-\mu t)\,t^r\mathrm{d}t=\frac{1}{\mu^{r+1}}\gamma(r+1,\mu x)$$

命题 2　对不完全 Gamma 函数 $\gamma(r+1,\mu x)$ 有：

$$\int_0^x\gamma(r+1,\mu t)\,\mathrm{d}t=\left[-\frac{1}{\mu}\sum_{l=0}^{r}\frac{\gamma(l+1,\mu x)}{l!}+x\right]r!,$$
$$r=0,1,2\cdots$$

命题 3　对不完全 Gamma 函数 $\gamma(r+1,\mu x)$，$r=0,1,2\cdots$，有：

$$\int_0^x\gamma(r+1,\mu t)\exp(-\lambda t)\,\mathrm{d}t=-\frac{1}{\lambda}\gamma(r+1,\mu x)\exp(-\lambda x)+\frac{1}{\lambda}\left(\frac{\mu}{\lambda+\mu}\right)^{r+1}\gamma(r+1,(\lambda+\mu)x)$$

下面由方程(3)出发继续迭代，并利用上述各命题得

$$P_0(x)=\left[\sum_{r=0}^{N-1}\frac{[(\lambda+\mu)/\mu]^{r+1}}{r!}P_{1+r}(0)\,\gamma(r+1,\mu x)+P_0(0)\right]\exp\left\{-\lambda x-\int_0^x\eta(u)\,\mathrm{d}u\right\}$$

$$P_{-n}(x)=\left\{\left(-\frac{\lambda}{\mu}\right)^n\sum_{l_0=0}^{N-1}\frac{(\lambda+\mu)^{l_0+1}}{\mu}P_{1+l_0}(0)\,C_{n,l_0}(x)+\sum_{r=0}^{n}P_{-n+r}(0)\,\frac{(\lambda x)^r}{r!}\right\}\exp\left\{-\lambda x-\int_0^x\eta(u)\,\mathrm{d}u\right\}$$

其中，$C_{n,l_0}(x)=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_n=0}^{l_{n-1}}\frac{\gamma(l_n+1,\mu x)}{l_n!}+\sum_{k=0}^{n}(-\mu)^k\frac{x^k}{k!}\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{n-k}=0}^{l_{n-k-1}}1,(1\leqslant n\leqslant M-1)$

$$P_{-M}(x)=\left\{\left(-\frac{\lambda}{\mu}\right)^{M-1}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,D_{l_0}(x)+\sum_{r=0}^{N-1}P_{-M+1+r}(0)\,\frac{\gamma(r+1,\lambda x)}{r!}\right\}\exp\left\{-\int_0^x\eta(u)\,\mathrm{d}u\right\}$$

其中，

$$D_{l_0}(x)=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1}=0}^{l_{M-2}}\frac{1}{l_{M-1}!}\left[-\gamma(l_{M-1}+1,\mu x)\exp(-\lambda x)+\left(\frac{\mu}{\lambda+\mu}\right)^{l_{M-1}+1}\gamma(l_{M-1}+1,(\lambda+\mu)x)\right]+\sum_{k=0}^{M-1}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1-k}=0}^{l_{M-2-k}}1\right)\left(-\frac{\mu}{\lambda}\right)^k\frac{\gamma(k+1,\lambda x)}{k!}$$

4　系统稳态概率及数量指标

记

$$H_r(\lambda)=\int_0^{+\infty}x^r\exp(-\lambda x)F(x)\,\mathrm{d}x \tag{10}$$

$$I_r(\lambda)=\int_0^{+\infty}\gamma(r+1,\lambda x)F(x)\,\mathrm{d}x \tag{11}$$

$$J_r(\mu,\lambda)=\int_0^{+\infty}\gamma(r+1,\mu x)\exp(-\lambda x)F(x)\,\mathrm{d}x \tag{12}$$

则

$$C_{n,l_0}=\int_0^{+\infty}C_{n,l_0}(x)\exp(-\lambda x)F(x)\,\mathrm{d}x=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_n=0}^{l_{n-1}}\frac{1}{l_n!}J_{l_n}(\mu,\lambda)+\sum_{k=0}^{n}(-\mu)^k\frac{1}{k!}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{n-k}=0}^{l_{n-k-1}}1\right)H_k(\lambda)$$

$$D_{l_0}=\int_0^{+\infty}D_{l_0}(x)F(x)\,\mathrm{d}x=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1}=0}^{l_{M-2}}\frac{1}{l_{M-1}!}[-J_{l_{M-1}}(\mu,\lambda)+$$

$$\left(\frac{\mu}{\lambda+\mu}\right)^{l_{M-1}+1} I_{l_{M-1}}(\lambda+\mu)] +$$

$$\sum_{k=0}^{M-1}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1-k}=0}^{l_{M-2-k}}1\right)\left(-\frac{\mu}{\lambda}\right)^{k}\frac{1}{k!}I_k(\lambda)$$

记 $P_n=\int_0^{+\infty}P_n(x)\,\mathrm{d}x$，$-M\leqslant n\leqslant N$，则

$$P_{-n}=\left(-\frac{\lambda}{\mu}\right)^{n}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,C_{n,l_0}+\sum_{r=0}^{n}P_{-n+r}(0)\,\frac{\lambda^r}{r!}H_r(\lambda),\quad(1\leqslant n\leqslant M-1)\tag{13}$$

$$P_{-M}=\left(-\frac{\lambda}{\mu}\right)^{M-1}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,D_{l_0}+\sum_{r=0}^{M-1}P_{-M+1+r}(0)\,\frac{1}{r!}I_r(\lambda)\tag{14}$$

$$P_0=\left[\sum_{r=0}^{N-1}\frac{[(\lambda+\mu)/\mu]^{r+1}}{r!}P_{1+r}(0)\,J_r(\mu,\lambda)+H_0(\lambda)\,P_0(0)\right]\tag{15}$$

$$P_n=\sum_{r=0}^{N-n}\frac{(\lambda+\mu)^r}{r!}P_{n+r}(0)\,H_r(\lambda+\mu),\quad(1\leqslant n\leqslant N)\tag{16}$$

至此，稳态概率 P_n 均已用显式表为 $P_n(0)$ 的线性函数。只要求出 $P_n(0)$，则可得到各指标，如：

①系统中有顾客的概率

$$\sum_{n=1}^{N}P_n=\sum_{n=1}^{N}\sum_{r=0}^{N-n}\frac{(\lambda+\mu)^r}{r!}P_{n+r}(0)\,H_r(\lambda+\mu)$$

②系统中有出租车的概率

$$\sum_{n=-1}^{-M}P_n=\sum_{n=-1}^{-M+1}\left[\left(-\frac{\lambda}{\mu}\right)^{n}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}\cdot P_{1+l_0}(0)\,C_{n,l_0}+\sum_{r=0}^{n}P_{-n+r}(0)\,\frac{\lambda^r}{r!}I_r(\lambda)\right]+\left[\left(-\frac{\lambda}{\mu}\right)^{M-1}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,D_{l_0}+\sum_{r=0}^{M-1}P_{-M+1+r}(0)\,\frac{1}{r!}I_r(\lambda)\right]$$

③系统空闲的概率

$$P_0=\left[\sum_{r=0}^{N-1}\frac{[(\lambda+\mu)/\mu]^{r+1}}{r!}\cdot P_{1+r}(0)\,J_r(\mu,\lambda)+H_0(\lambda)\,P_0(0)\right]$$

④服务端平均队长

$$L_s=\sum_{n=1}^{M}nP_{-n}=\sum_{n=1}^{M-1}n\left[\left(-\frac{\lambda}{\mu}\right)^{n}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,C_{n,l_0}+\sum_{r=0}^{n}P_{-n+r}(0)\,\frac{\lambda^r}{r!}H_r(\lambda)\right]+M\left[\left(-\frac{\lambda}{\mu}\right)^{M-1}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,D_{l_0}+\sum_{r=0}^{M-1}P_{-M+1+r}(0)\,\frac{1}{r!}I_r(\lambda)\right]$$

⑤顾客端平均队长

$$L_c=\sum_{n=1}^{N}nP_n=\sum_{n=1}^{N}n\sum_{r=0}^{N-n}\frac{(\lambda+\mu)^r}{r!}P_{n+r}(0)\,H_r(\lambda+\mu)$$

5 求解 $P_n(0)$

记

$$K_r(\lambda)=\int_0^{+\infty}x^r\exp(-\lambda x)f(x)\,\mathrm{d}x\tag{17}$$

$$L_r(\lambda)=\int_0^{+\infty}\gamma(r+1,\lambda x)f(x)\,\mathrm{d}x\tag{18}$$

$$M_r(\mu,\lambda)=\int_0^{+\infty}\gamma(r+1,\mu x)\exp(-\lambda x)f(x)\,\mathrm{d}x\tag{19}$$

则

$$C_{n,l_0}=\int_0^{+\infty}C_{n,l_0}(x)\exp(-\lambda x)f(x)\,\mathrm{d}x=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_n=0}^{l_{n-1}}\frac{1}{l_n!}M_{l_n}(\mu,\lambda)+\sum_{k=0}^{n}(-\mu)^k\frac{1}{k!}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{n-k}=0}^{l_{n-k-1}}1\right)K_k(\lambda)\tag{20}$$

$$D_{l_0}=\int_0^{+\infty}D_{l_0}(x)f(x)\,\mathrm{d}x=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1}=0}^{l_{M-2}}\frac{1}{l_{M-1}!}\left[-M_{l_{M-1}}(\mu,\lambda)+\left(\frac{\mu}{\lambda+\mu}\right)^{l_{M-1}+1}L_{l_{M-1}}(\lambda+\mu)\right]+\sum_{k=0}^{M-1}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1-k}=0}^{l_{M-2-k}}1\right)\left(-\frac{\mu}{\lambda}\right)^{k}\frac{1}{k!}L_k(\lambda)\tag{21}$$

从而

$$\int_0^{+\infty}\eta(x)\,P_{-n}(x)\,\mathrm{d}x=\left(-\frac{\lambda}{\mu}\right)^{n}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)\,C_{n,l_0}+\sum_{r=0}^{n}P_{-n+r}(0)\,\frac{\lambda^r}{r!}K_r(\lambda),\ (1\leqslant n\leqslant M-1)$$

$$\int_0^{+\infty}\eta(x)\,P_{-M}(x)\,\mathrm{d}x=\left(-\frac{\lambda}{\mu}\right)^{M-1}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)D_{l_0}+\sum_{r=0}^{M-1}P_{-M+1+r}(0)\,\frac{1}{r!}L_r(\lambda)$$

$$\int_0^{+\infty}\eta(x)\,P_0(x)\,\mathrm{d}x=\left[\sum_{r=0}^{N-1}\frac{[(\lambda+\mu)/\mu]^{r+1}}{r!}\cdot P_{1+r}(0)\,M_r(\mu,\lambda)+P_0(0)\,K_0(\lambda)\right]$$

$$\int_0^{+\infty}\eta(x)P_n(x)\,dx=\sum_{r=0}^{N-n}\frac{(\lambda+\mu)^r}{r!}P_{n+r}(0)K_r(\lambda+\mu),\quad(1\leqslant n\leqslant N)$$

注意到 $\int_0^{+\infty}\eta(x)P_n(x)\,dx$ 和 $P_n(n=-M,-M+1,\cdots,-1,0,1,2,\cdots,N)$ 都是 $P_n(0)$ 的线性函数，将它们分别代入(7)～(9)式和归一化方程 $\sum_{n=-M}^{N}P_n=1$，得关于 $P_n(0)(-M+1\leqslant n\leqslant N)$ 的非齐次线性方程组，从而可解出 $P_n(0)$。

6　顾客到达时间间隔是参数为 (β,k) 的 Γ 分布且 $S=1$ 的情形

$S=1$ 时，边界条件(7)～(9)式化为

$$P_n(0)=\int_0^{+\infty}\eta(x)P_{n-1}(x)\,dx,\quad(-M+1\leqslant n\leqslant N-1)\tag{22}$$

$$P_N(0)=\int_0^{+\infty}\eta(x)[P_{N-1}(x)+P_N(x)]\,dx\tag{23}$$

进一步假设顾客到达时间间隔服从参数为 (β,k) 的 Γ 分布，即有分布密度

$$f(x)=\frac{\beta^k x^{k-1}\exp(-\beta x)}{\Gamma(k)}$$

这时(10)～(12)和(17)～(19)式分别为

$$H_{r,k}(\lambda,\beta)=\int_0^{+\infty}x^r\exp(-\lambda x)F(x)\,dx=\frac{1}{\Gamma(k)}\int_0^{+\infty}x^r\exp(-\lambda x)\Gamma(k,\beta x)\,dx=\frac{r!}{\lambda^{r+1}}-\frac{r!}{\lambda^{r+1}\Gamma(k)}\left(\frac{\lambda}{\lambda+\beta}\right)^k\sum_{i=0}^{r}\left(\frac{\lambda}{\lambda+\beta}\right)^i\frac{\Gamma(k+i)}{i!}$$

$$I_{r,k}(\lambda,\beta)=\int_0^{+\infty}\gamma(r+1,\lambda x)F(x)\,dx=-\lambda^{r+1}H_{r+1,k}(\lambda,\beta)+\frac{\Gamma(r+1)\Gamma(k+1)}{\beta\Gamma(k)}-\frac{\beta^k\Gamma(r+1)}{\Gamma(k)}H_{k,r+1}(\beta,\lambda)$$

$$K_{r,k}(\lambda,\beta)=\int_0^{+\infty}x^r\exp(-\lambda x)f(x)\,dx=\frac{\beta^k\Gamma(r+k)}{(\lambda+\beta)^{r+k}\Gamma(k)}$$

$$L_{r,k}(\lambda,\beta)=\int_0^{+\infty}\gamma(r+1,\lambda x)f(x)\,dx=\Gamma(r+1)-\frac{\beta^k\Gamma(r+1)}{\Gamma(k)}H_{k-1,r+1}(\beta,\lambda)$$

$$M_{r,k}(\mu,\lambda,\beta)=\int_0^{+\infty}\gamma(r+1,\mu x)e^{-\lambda x}f(x)\,dx=\frac{\beta^k\Gamma(r+1)}{(\lambda+\beta)^k}-\frac{\beta^k\Gamma(r+1)}{\Gamma(k)}H_{k-1,r+1}(\lambda+\beta,u)$$

$$J_{r,k}(\mu,\lambda,\beta)=\int_0^{+\infty}\gamma(r+1,\mu x)e^{-\lambda x}F(x)\,dx=\frac{1}{\lambda}[\mu^{r+1}H_{r,k}(\lambda+\mu,\beta)-M_{r,k}(\mu,\lambda,\beta)]$$

从而(20)和(21)式为

$$C_{n,l_0}=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_n=0}^{l_{n-1}}\frac{1}{l_n!}M_{l_n,k}(\mu,\lambda,\beta)+\sum_{i=0}^{n}(-\mu)^i\frac{1}{i!}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{n-i}=0}^{l_{n-i-1}}1\right)K_{i,k}(\lambda,\beta)$$

$$D_{l_0}=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1}=0}^{l_{M-2}}\frac{1}{l_{M-1}!}\left[-M_{l_{M-1},k}(\mu,\lambda,\beta)+\left(\frac{\mu}{\lambda+\mu}\right)^{l_{M-1}+1}L_{l_{M-1},k}((\lambda+\mu),\beta)\right]+\sum_{k=0}^{M-1}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1-k}=0}^{l_{M-2-k}}1\right)\left(-\frac{\mu}{\lambda}\right)^i\frac{1}{i!}L_{i,k}(\lambda,\beta)$$

代入(22)和(23)式得关于 $P_n(0)(-M+1\leqslant n\leqslant N)$ 的线性方程组：

$$P_{-n+1}(0)=\left(-\frac{\lambda}{\mu}\right)^n\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}P_{1+l_0}(0)C_{n,l_0}+\sum_{r=0}^{n}P_{-n+r}(0)\frac{\lambda^r}{r!}K_{r,k}(\lambda,\beta),\quad(1\leqslant n\leqslant M-1)$$

$$P_{-M+1}(0)=\left(-\frac{\lambda}{\mu}\right)^{M-1}\sum_{l_0=0}^{N-1}\left(\frac{\lambda+\mu}{\mu}\right)^{l_0+1}。P_{1+l_0}(0)D_{l_0}+\sum_{r=0}^{M-1}P_{-M+1+r}(0)\frac{1}{r!}L_{r,k}(\lambda,\beta)$$

$$P_1(0)=\sum_{r=0}^{N-1}\frac{[(\lambda+\mu)/\mu]^{r+1}}{r!}。P_{1+r}(0)M_{r,k}(\mu,\lambda,\beta)+P_0(0)K_{0,k}(\lambda,\beta)$$

$$P_{n+1}(0)=\sum_{r=0}^{N-n}\frac{(\lambda+\mu)^r}{r!}P_{n+r}(0)K_{r,k}(\lambda+\mu,\beta),\quad(1\leqslant n\leqslant N-2)$$

$$P_N(0)=\sum_{r=0}^{1}\frac{(\lambda+\mu)^r}{r!}P_{N-1+r}(0)K_{r,k}(\lambda+\mu,\beta)+P_N(0)K_{0,k}(\lambda+\mu,\beta)$$

将它们与归一化条件 $\sum_{n=-M}^{N}P_n=1$ 联立可解出 $P_n(0)$。这时进一步有

$$C_{n,l_0}=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_n=0}^{l_{n-1}}\frac{1}{l_n!}J_{l_n,k}(\mu,\lambda,\beta)+\sum_{i=0}^{n}(-\mu)^i\frac{1}{i!}\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{n-i}=0}^{l_{n-i-1}}1\right)H_{i,k}(\lambda,\beta)$$

$$D_{l_0}=\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1}=0}^{l_{M-2}}\frac{1}{l_{M-1}!}。[-J_{l_{M-1},k}(\mu,\lambda,\beta)+$$

$$\left(\frac{\mu}{\lambda+\mu}\right)^{l_{M-1}+1} I_{l_{M-1},k}((\lambda+\mu),\beta)+$$

$$\sum_{i=0}^{M-1}\left[\left(\sum_{l_1=0}^{l_0}\sum_{l_2=0}^{l_1}\cdots\sum_{l_{M-1-i}=0}^{l_{M-2-i}}1\right)\left(-\frac{\mu}{\lambda}\right)^{i}\frac{1}{i\,!}I_{i,k}(\lambda,\beta)\right]$$

代入(13)~(16)式得相应的 $P_n(-M+1\leqslant n\leqslant N)$。

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Double-ended Queue System with Negative Customers

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Abstract: The double-ended queue system with negative customers and limited waiting space for both ends is discussed under the assumption that the interarrival time of customer end is taken as general distribution in batch and of sever end as Poission. Using the supplementary variable technique, the stable probability of the system is obtained recursively. The case that the arrival of customer is single and with Γ-distribution is considered as particular.

Key words: bouble-ended queue; negative customer; batch arrival; supplementary variable technique