

# 分数阶整数阶多混沌系统的自适应滑模同步\*

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**摘要:** 利用自适应滑模方法研究具有不确定性和外扰下一类分数阶整数阶多混沌系统的同步。通过设计滑模函数及控制律, 获得整数阶及分数阶多混沌系统自适应滑模同步的充分条件, 并用数值仿真对所取得的结果进行了验证。

**关键词:** 自适应; 高阶; 滑模; 多混沌

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## Self-adaptive sliding mode synchronization of fractional-order and integer-order uncertain multi-chaotic systems

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**Abstract:** In this paper, we study synchronization of a class of fractional-order and integer-order multi-chaotic systems with uncertainties models and external disturbances using self-adaptive sliding mode approach. And the sufficient conditions were arrived for fractional-order and integer-order multi-chaotic systems getting self-adaptive sliding mode synchronization by design sliding mode functions and control laws. Numerical simulation demonstrate the correctness of the conclusion.

**Key words:** self-apaptive; high-order; sliding mode; multi-chaotic

混沌控制引起了控制领域众多专家的高度关注<sup>[1-6]</sup>。混沌系统对初始条件和系统参数变化异常敏感, 它的同步问题在保密通信、信号与系统、声学、电磁、生化系统等领域得到了广泛的应用。随着分数阶微积分的发展, 分数阶系统的滑模控制方法被相继提出<sup>[7-15]</sup>, 例如: 文献 [16] 研究分数阶 Duffling 系统的新型滑模趋近律。文献 [17] 研究分数阶 Genesio-Tesi 系统的适应转移滑模同步。文献 [18] 研究分数阶 Duffling 系统的终端滑模同步控制。文献 [14] 研究分数阶 Newton-Leipnik 系统的滑模扰动观测器同步。文献 [19] 研究不确定分数阶系统有限时间同步的充分条件。文献 [20] 研究分数阶非线性系统的滑模同步。本文利用自适应滑模方法研究具有不确定性和外扰下的一类分数阶整数阶多混沌系统的同步。通过设计滑模函数及控制律, 获得了整数阶分数阶多混沌系统取得自适应滑模同步的充分性条件。

## 1 主要结果

定义 1<sup>[21]</sup>

连续函数  $x(t)$  的  $q$  阶 Caputo 分数阶导数定义为

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$${}_0^c D_{t_0,t}^q x(t) = D_{t_0,t}^{-(n-q)} \frac{d^n}{dt^n} x(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^t (t-\tau)^{n-q-1} x^{(n)}(\tau) d\tau, \quad n-1 < q < n \in \mathbf{Z}^+$$

分数阶多混沌系统建模为

$$\begin{cases} D_t^q x_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + f_1(x, t), \\ D_t^q x_2 = a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + f_2(x, t), \\ \vdots \\ D_t^q x_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + f_n(x, t) \end{cases} \quad (1)$$

其中,  $q \in (0, 1), x = [x_1, x_2, \dots, x_n]^T, f_i(x, t)$  为非线性函数。以 (1) 为主系统, 设计从系统为

$$\begin{cases} D_t^q y_1 = a_{11}y_1 + a_{12}y_2 + \dots + a_{1n}y_n + f_1(y, t) + \Delta f_1(y, t) + d_1(t) + u_1(t), \\ D_t^q y_2 = a_{21}y_1 + a_{22}y_2 + \dots + a_{2n}y_n + f_2(y, t) + \Delta f_2(y, t) + d_2(t) + u_2(t), \\ \vdots \\ D_t^q y_n = a_{n1}y_1 + a_{n2}y_2 + \dots + a_{nn}y_n + f_n(y, t) + \Delta f_n(y, t) + d_n(t) + u_n(t) \end{cases} \quad (2)$$

其中,  $y = [y_1, y_2, \dots, y_n]^T, \Delta f_i(y, t)$  为不确定项,  $d(t)$  为外部扰动。

**假设 1**

设不确定项  $\Delta f_i(y, t)$  和外部扰动  $d_i(t)$  有界, 即存在未知参数  $m_i, n_i > 0$  使得

$$|\Delta f_i(y, t)| < m_i, \quad |d_i(t)| < n_i, \quad i = 1, 2, \dots, n$$

定义  $e_i(t) = y_i(t) - x_i(t), i = 1, 2, \dots, n$ , 得系统

$$\begin{cases} D_t^q e_1 = a_{11}e_1 + a_{12}e_2 + \dots + a_{1n}e_n + f_1(y, t) - f_1(x, t) + \Delta f_1(y, t) + d_1(t) + u_1(t), \\ D_t^q e_2 = a_{21}e_1 + a_{22}e_2 + \dots + a_{2n}e_n + f_2(y, t) - f_2(x, t) + \Delta f_2(y, t) + d_2(t) + u_2(t), \\ \vdots \\ D_t^q e_n = a_{n1}e_1 + a_{n2}e_2 + \dots + a_{nn}e_n + f_n(y, t) - f_n(x, t) + \Delta f_n(y, t) + d_n(t) + u_n(t) \end{cases} \quad (3)$$

**引理 1**<sup>[21]</sup>

若  $x(t)$  为连续可微的函数, 则对任意的  $t \geq 0$  有

$$\frac{1}{2} {}_0 D_t^\alpha x^T(t)x(t) \leq x^T(t) {}_0 D_t^\alpha x(t), \quad \forall \alpha \in (0, 1)$$

**引理 2**<sup>[21]</sup>

设  $V(t) = \frac{1}{2} (y_1^2(t) + y_2^2(t))$ , 其中  $y_1(t), y_2(t) \in \mathbf{R}$  具有连续一阶导数, 若存在常数  $k > 0$ , 使得  ${}_0 D_t^\alpha V(t) \leq -k y_1^2(t)$ , 则  $\|y_1(t)\|, \|y_2(t)\|$  有界且  $y_1^2(t) \leq 2V(0)E_{\alpha,1}(-2kt^\alpha)$ 。其中,  $E_{\alpha,\beta}(\cdot)$  表示双参数 Mittag-Leffler 函数。

**注**

若  $y_1^2(t) \leq 2V(0)E_{\alpha,1}(-2kt^\alpha)$ , 则由文献 [12] 可知:  $y_1(t)$  是 Mittag-Leffler 稳定的, 且  $\lim_{t \rightarrow \infty} \|y_1(t)\| = 0$ 。

**定理 1**

满足假设 1, 设计滑模面  $s_i(t) = e_i(t) + \lambda_i D_t^{-q} e_i(t)$ , 控制器

$$u_i(t) = -\lambda_i e_i - (a_{i1}e_1 + a_{i2}e_2 + \dots + a_{in}e_n) + f_i(x, t) - f_i(y, t) - (\hat{m}_i + \hat{n}_i) \text{sgn}(s_i) - k_i |s_i| \text{sgn}(s_i),$$

自适应律

$$\begin{cases} D_t^q \hat{m}_i = |s_i|, \hat{m}_i(0) = \hat{m}_{i0}, \\ D_t^q \hat{n}_i = |s_i|, \hat{n}_i(0) = \hat{n}_{i0}, \end{cases} \quad i = 1, 2, \dots, n$$

式中,  $\hat{m}_i, \hat{n}_i$  分别为  $m_i, n_i$  的估计值,  $k_i > 0, i = 1, 2, \dots, n$ , 则 (1) 与 (2) 达到自适应滑模同步。

**证明**

滑模面上满足  $s_i(t) = 0, \dot{s}_i(t) = 0$ , 则  $D_t^q s_i(t) = D_t^q e_i(t) + \lambda_i e_i(t) = 0$ ,

因为  $D_i^q e_i(t) = -\lambda_i e_i(t)$ , 从而  $\lim_{t \rightarrow \infty} e_i(t) = 0$ , 所以该滑模面稳定。

不在滑模面上设计  $V(t) = \sum_{i=1}^n \left\{ \frac{1}{2} s_i^2(t) + \frac{1}{2} (\hat{m}_i - m_i)^2 + \frac{1}{2} (\hat{n}_i - n_i)^2 \right\}$ , 根据引理 1 有

$$\begin{aligned} D_i^q V &\leq \left\{ s_i(t) [D_i^q e_i(t) + \lambda_i e_i(t)] + (\hat{m}_i - m_i) |s_i| + (\hat{n}_i - n_i) |s_i| \right\} \\ &= \sum_{i=1}^n \left\{ s_i(t) [a_{i1} e_1 + a_{i2} e_2 + \dots + a_{in} e_n + f_i(y, t) - f_i(x, t) + \Delta f_i(y, t) + d_i(t) + u_i(t) \right. \\ &\quad \left. + \lambda_i e_i(t)] + (\hat{m}_i - m_i) |s_i| + (\hat{n}_i - n_i) |s_i| \right\} \\ &\leq \sum_{i=1}^n \left\{ |s_i| [(m_i + n_i) - (\hat{m}_i + \hat{n}_i) - k_i |s_i|] + (\hat{m}_i - m_i) |s_i| + (\hat{n}_i - n_i) |s_i| \right\} \\ &= -\sum_{i=1}^n k_i |s_i|^2 < 0, \end{aligned}$$

利用引理 2, 得到  $\lim_{t \rightarrow \infty} s_i(t) = 0 \Rightarrow e_i(t) \rightarrow 0$ 。

整数阶多混沌系统建模为

$$\begin{cases} \dot{x}_1 = a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n + f_1(x, t), \\ \dot{x}_2 = a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n + f_2(x, t), \\ \vdots \\ \dot{x}_n = a_{n1} x_1 + a_{n2} x_2 + \dots + a_{nn} x_n + f_n(x, t) \end{cases} \quad (4)$$

其中,  $q \in (0, 1)$ ,  $x = [x_1, x_2, \dots, x_n]^T$ ,  $f_i(x, t)$  为非线性函数。以 (4) 为主系统, 设计从系统为

$$\begin{cases} \dot{y}_1 = a_{11} y_1 + a_{12} y_2 + \dots + a_{1n} y_n + f_1(y, t) + \Delta f_1(y, t) + d_1(t) + u_1(t), \\ \dot{y}_2 = a_{21} y_1 + a_{22} y_2 + \dots + a_{2n} y_n + f_2(y, t) + \Delta f_2(y, t) + d_2(t) + u_2(t), \\ \vdots \\ \dot{y}_n = a_{n1} y_1 + a_{n2} y_2 + \dots + a_{nn} y_n + f_n(y, t) + \Delta f_n(y, t) + d_n(t) + u_n(t) \end{cases} \quad (5)$$

其中,  $y = [y_1, y_2, \dots, y_n]^T$ ,  $\Delta f_i(y, t)$  为不确定项,  $d(t)$  为外扰。

定义  $e_i(t) = y_i(t) - x_i(t)$ ,  $i = 1, 2, \dots, n$ , 得系统

$$\begin{cases} \dot{e}_1 = a_{11} e_1 + a_{12} e_2 + \dots + a_{1n} e_n + f_1(y, t) - f_1(x, t) + \Delta f_1(y, t) + d_1(t) + u_1(t), \\ \dot{e}_2 = a_{21} e_1 + a_{22} e_2 + \dots + a_{2n} e_n + f_2(y, t) - f_2(x, t) + \Delta f_2(y, t) + d_2(t) + u_2(t), \\ \vdots \\ \dot{e}_n = a_{n1} e_1 + a_{n2} e_2 + \dots + a_{nn} e_n + f_n(y, t) - f_n(x, t) + \Delta f_n(y, t) + d_n(t) + u_n(t) \end{cases} \quad (6)$$

**引理 3** (Barbarlet's 引理)

若函数  $f(t)$  在  $[0, +\infty)$  上一致连续, 并且广义积分  $\int_0^{+\infty} f(t) dt$  存在, 则有  $\lim_{t \rightarrow \infty} f(t) = 0$ 。其中,  $f(t)$  表示一致连续的函数。

**定理 2**

满足假设 1, 设计滑模面

$$s_i(t) = e_i(t) + \lambda_i \int_0^t e_i(\tau) d\tau,$$

$$u_i(t) = -\lambda_i e_i - (a_{i1} e_1 + a_{i2} e_2 + \dots + a_{in} e_n) + f_i(x, t) - f_i(y, t) - (\hat{m}_i + \hat{n}_i) \operatorname{sgn}(s_i) - k_i |s_i| \operatorname{sgn}(s_i),$$

自适应律

$$\begin{cases} \dot{\hat{m}}_i = |s_i|, \hat{m}_i(0) = \hat{m}_{i0}, \\ \dot{\hat{n}}_i = |s_i|, \hat{n}_i(0) = \hat{n}_{i0}, \end{cases} \quad i = 1, 2, \dots, n$$

式中,  $\hat{m}_i, \hat{n}_i$  分别为  $m_i, n_i$  的估计值,  $k_i > 0$ , 则 (4) 与 (5) 达到自适应滑模同步。

**证明**

滑模面上时, 满足  $s_i(t) = 0, \dot{s}_i(t) = 0$ , 从而有

$$\dot{e}_i(t) + \lambda_i e_i(t) = 0,$$

所以  $\lim_{t \rightarrow \infty} e_i(t) = 0$ 。故该滑模面稳定。

不在滑模面上设计函数  $V(t) = \sum_{i=1}^n \left\{ \frac{1}{2} s_i^2(t) + \frac{1}{2} (\hat{m}_i - m_i)^2 + \frac{1}{2} (\hat{n}_i - n_i)^2 \right\}$ , 得到

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n \left\{ s_i(t) [\dot{e}_i(t) + \lambda_i e_i(t)] + (\hat{m}_i - m_i) |s_i| + (\hat{n}_i - n_i) |s_i| \right\} \\ &= \sum_{i=1}^n \left\{ s_i(t) [a_{i1} e_1 + a_{i2} e_2 + \dots + a_{in} e_n + f_i(y, t) - f_i(x, t) + \Delta f_i(y, t) + d_i(t) + u_i(t) + \lambda_i e_i(t)] \right. \\ &\quad \left. + (\hat{m}_i - m_i) |s_i| + (\hat{n}_i - n_i) |s_i| \right\} \\ &\leq \sum_{i=1}^n \left\{ |s_i| [(m_i + n_i) - (\hat{m}_i + \hat{n}_i) - k_i |s_i|] + (\hat{m}_i - m_i) |s_i| + (\hat{n}_i - n_i) |s_i| \right\} \\ &= -\sum_{i=1}^n k_i |s_i|^2 < 0 \end{aligned}$$

两边积分  $\int_0^t \min \{k_i\} \sum_{i=1}^n |s_i(\tau)|^2 d\tau \leq -\int_0^t \dot{V} dt \leq V(0) - V(t) < V(0) < +\infty$ ,

利用引理3, 得到  $\lim_{t \rightarrow \infty} s_i(t) = 0 \Rightarrow e_i(t) \rightarrow 0$ 。

## 2 数值仿真

定理1以分数阶 Victor-Carmen 系统作为主系统。

$$\begin{cases} D_t^q x_1 = -x_1 - \alpha x_2 x_3, \\ D_t^q x_2 = -x_2 + cx_3 - \beta x_1 x_3, \\ D_t^q x_3 = -bx_1 - cx_2 + x_3 + \gamma x_1 x_2 \end{cases}$$

其中,  $x_1, x_2, x_3 \in \mathbf{R}^3$  系统状态,  $\alpha = 50, \beta = 20, \gamma = 4.1, c = 5, b = 9, q = 0.893$ 。

从系统设计为

$$\begin{cases} D_t^q y_1 = -y_1 - \alpha y_2 y_3 + \Delta f_1(y, t) + d_1(t) + u_1, \\ D_t^q y_2 = -y_2 + cy_3 - \beta y_1 y_3 + \Delta f_2(y, t) + d_2(t) + u_2, \\ D_t^q y_3 = -by_1 - cy_2 + y_3 + \gamma y_1 y_2 + \Delta f_3(y, t) + d_3(t) + u_3 \end{cases}$$

定义  $e_i(t) = y_i(t) - x_i(t), i = 1, 2, 3$ , 得系统

$$\begin{cases} D_t^q e_1 = -e_1 - \alpha y_2 y_3 + \alpha x_2 x_3 + \Delta f_1(y, t) + d_1(t) + u_1(t), \\ D_t^q e_2 = -e_2 + ce_3 - \beta y_1 y_3 + \beta x_1 x_3 + \Delta f_2(y, t) + d_2(t) + u_2(t), \\ D_t^q e_3 = -be_1 - ce_2 + e_3 + \gamma y_1 y_2 - \gamma x_1 x_2 + \Delta f_3(y, t) + d_3(t) + u_3(t) \end{cases}$$

设计滑模面  $s_i(t) = e_i(t) + \lambda_i D_t^{-q} e_i(t), f_i(y, t) + d_i(t) = 0.1y_3 \cos t + 0.1 \sin t$ ,

$$u_i(t) = -\lambda_i e_i - (a_{i1} e_1 + a_{i2} e_2 + a_{i3} e_3) + f_i(x, t) - f_i(y, t)$$

$$-(\hat{m}_i + \hat{n}_i) \operatorname{sgn}(s_i) - k_i |s_i| \operatorname{sgn}(s_i),$$

$$i = 1, 2, 3$$

$$\lambda_1 = 1.5, \lambda_2 = 1.5, \lambda_3 = 2, \hat{m}_i = 2, \hat{n}_i = 1.5, k_i = 2.5$$

定理2以整数阶 Victor-Carmen 系统作为主系统。

$$\begin{cases} \dot{x}_1 = -x_1 - \alpha x_2 x_3, \\ \dot{x}_2 = -x_2 + cx_3 - \beta x_1 x_3, \\ \dot{x}_3 = -bx_1 - cx_2 + x_3 + \gamma x_1 x_2 \end{cases}$$

从系统设计为

$$\begin{cases} \dot{y}_1 = -y_1 - \alpha y_2 y_3 + \Delta f_1(y, t) + d_1(t) + u_1, \\ \dot{y}_2 = -y_2 + cy_3 - \beta y_1 y_3 + \Delta f_2(y, t) + d_2(t) + u_2, \\ \dot{y}_3 = -by_1 - cy_2 + y_3 + \gamma y_1 y_2 + \Delta f_3(y, t) + d_3(t) + u_3 \end{cases}$$

定义  $e_i(t) = y_i(t) - x_i(t), i = 1, 2, 3$ , 得系统

$$\begin{cases} \dot{e}_1 = -e_1 - \alpha y_2 y_3 + \alpha x_2 x_3 + \Delta f_1(y, t) + d_1(t) + u_1(t), \\ \dot{e}_2 = -e_2 + c e_3 - \beta y_1 y_3 + \beta x_1 x_3 + \Delta f_2(y, t) + d_2(t) + u_2(t), \\ \dot{e}_3 = -b e_1 - c e_2 + e_3 + \gamma y_1 y_2 - \gamma x_1 x_2 + \Delta f_3(y, t) + d_3(t) + u_3(t) \end{cases}$$

设计滑模面  $s_i(t) = e_i(t) + \lambda_i \int_0^t e_i(\tau) d\tau$ ,  $f_i(y, t) + d_i(t) = 0.1y_3 \cos t + 0.1\sin t$ ,  $u_i(t) = -\lambda_i e_i - (a_{i1}e_1 + a_{i2}e_2 + a_{i3}e_3) + f_i(x, t) - f_i(y, t) - (\hat{m}_i + \hat{n}_i)\text{sgn}(s_i) - k_i |s_i| \text{sgn}(s_i)$ ,  $i = 1, 2, 3$ ,  $\lambda_1 = 2.5, \lambda_2 = 2$ ,  $\hat{n}_i = 2$ ,  $k_i = 2$ 。定理 1、2 中的误差如图 1-2 所示, 从中可看到初始时刻误差相差较大, 距坐标原点较远, 随着时间的推移, 逐渐趋向一致且逐渐向原点靠近, 表明整数阶、分数阶 Victor-Carmen 系统系统取得了混沌同步。

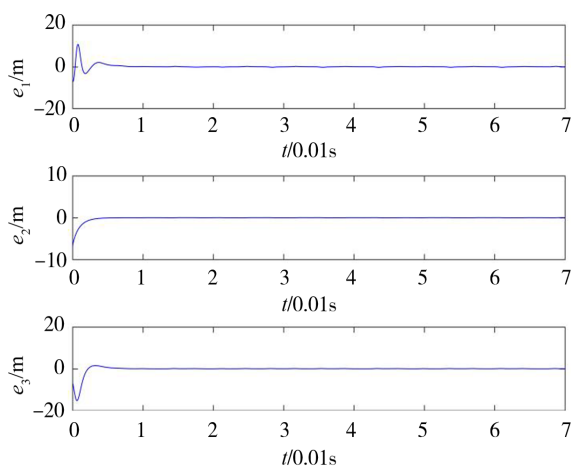


图1 分数阶 Victor-Carmen 系统误差

Fig. 1 Fractional-order Victor-Carmen system errors

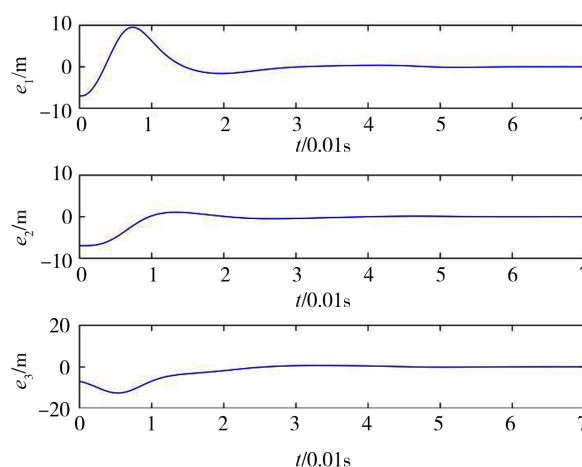


图2 整数阶 Victor-Carmen 系统误差

Fig. 2 Integer-order Victor-Carmen system errors

### 3 结 论

研究分数阶整数阶具有不确定性和外扰下多混沌系统的自适应滑模同步, 设计了滑模控制器和自适应规则, 得到了分数阶及整数阶多混沌系统取得自适应滑模同步的充分条件, 数值仿真检验了所得结果的正确性。

#### 参考文献:

- [1] LIU Y, GUO B, PARK J, et al. Non-fragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control[J]. IEEE Trans Neural Network Learn Systems 2016. DOI: 10.1109/TNNLS.2016.26(8):709-716.
- [2] HAN X, WU H, FANG B. Adaptive exponential synchronization of memristive neural networks with mixed time-varying delays [J]. Neurocomputing, 2016, 20(3):40-50.
- [3] CHEN C, LI L, PENG H, et al. Finite-time synchronization of memristor-based neural networks with mixed delays [J]. Neurocomputing, 2017, 235(16):83-89.
- [4] TAO L, CHEN Q, NAN Y. Disturbance-observer based adaptive control for second-order nonlinear systems using chattering-free reaching law[J]. International Journal of Control, Automation and Systems, 2019, 17(2):356-369.
- [5] LIN C. A four-dimensional hyperchaotic finance system and its control problems[J]. Journal of Control Science & Engineering, 2018, 23(6): 1-12.
- [6] YANG Y G, XU W, SUN Y H, et al. Stochastic bifurcations in the nonlinear vibroimpact system with fractional derivative under random excitation[J]. Communications in Nonlinear Science & Numerical Simulation, 2017, 42(6): 62-72.
- [7] WU Z, SHI P, SU H, et al. Exponential synchronization of neural networks with discrete and distributed delays under time-varying sampling[J]. IEEE Trans Neural Netw Learn Syst, 2012, 23(6):1368-1376.

- [8] LIU Y, GUO B, PARK J, et al. Nonfragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control[J]. IEEE Trans Neural Netw Learn Syst 2016. DOI: 10.1109/TNNLS.2016.26(8):709-716.
- [9] HAN X, WU H, FANG B. Adaptive exponential synchronization of memristive neural networks with mixed time-varying delays[J]. Neurocomputing, 2016,20(3):40-50.
- [10] CHEN C, LI L, PENG H, et al. Finite-time synchronization of memristor-based neural networks with mixed delays [J]. Neurocomputing, 2017,235(16):83-89.
- [11] HAMAMC I, KOLKSAL M. Calculation of all stabilizing fractional-order PD controllers for integrating time delay systems. [J]. Comput Math Appl, 2010,15(2):1267-1278.
- [12] MATOUK A. Chaos feedback and synchronization of fractional-order modified autonomous Van der pol-Duffling circuit [J]. Commun Nonlinear Sci Numer Simul, 2011,16(5):975-986.
- [13] SHAHIRI M, GHADRI R, RANJBAR N, et al. Chaotic fractional-order coullet system: synchronization and control approach [J]. Commun Nonlinear Sci Numer Simul, 2010,15(6):665-674.
- [14] MAO B X. Four methods for sliding mode synchronization of fractional-order new hyperchaotic system [J]. Paper Asia, 2018, 12(5):118-122.
- [15] 付景超,张中华. 超混沌 Bao 系统状态线性反馈控制及自适应控制[J]. 控制与决策,2016,31(9):1707-1710.  
FU J C, ZHANG Z H. Linear state feedback control and adaptive backstepping control of hyperchaotic Bao system [J]. Control and Decision, 2016,31(9):1707-1710.
- [16] 毛北行,李巧利. 一类分数阶 Genesio-Tesi 系统的滑模混沌同步[J]. 中山大学学报(自然版),2017,56(2):76-79.  
MAO B X, LI Q L. Sliding mode chaos synchronization of a class of fractional-order Genesio-Tesi systems [J]. Acta Scientiarum Naturalium Universitatis Sunyatseni, 2017,56(2):76-79.
- [17] 毛北行. 基于一种新型趋近律的分数阶 Duffling 不确定系统的自适应滑模同步[J]. 中山大学学报(自然科学版),2017, 56(4):64-67.  
MAO B X. Self-adaptive sliding mode synchronization of fractional-order uncertainty Duffling systems based on a new reaching law [J]. Acta Scientiarum Naturalium Universitatis Sunyatseni, 2017,56(4):64-67.
- [18] 毛北行. 分数阶 Genesio-Tesi 混沌系统的适应转移函数滑模同步方法[J]. 南京理工大学学报,2018,42(5):586-590.  
MAO B X. Adaptive transfer function sliding mode synchronization method of fractional order Genesio-Tesi chaotic system [J]. Journal of Nanjing university of Science and Technology, 2018,42(5):586-590.
- [19] 毛北行,周长芹. 分数阶不确定 Duffling 混沌系统的终端滑模同步[J]. 东北师范大学学报(自然科学版),2018,50(2): 47-50.  
MAO B X, ZHOU C Q. Terminal sliding mode synchronization of fractional-order uncertainty Duffling chaotic systems [J]. Journal of Northeast Normal University(Natural Science Edition), 2018,50(2):47-50.
- [20] KHANAYU B, TYAGIART I. Fractional order disturbance observer based adaptive sliding mode hybrid projective synchronization of fractional order Newton-Leipnik chaotic system [J]. International Journal Dynamics Control, 2018, 53(6):1139-1149.
- [21] 闫丽宏. 广义分数阶 Sprott-C 混沌系统的有限时间滑模同步[J]. 吉林学报(理学版),2019,57(4):940-945.  
YAN L H. Finite-time sliding mode synchronization of generalized fractional-order Sprott-C Chaotic systems [J]. Journal of Jilin University(Science Edition), 2019,57(4):940-945.

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